

Trigonometry:
WITH AN
INTRODUCTION
TO THE
Use of Both Globes,
AND
Projection of the SPHERE *in Plano.*

To which is subjoin'd,
An APPENDIX
Applying the DOCTRINE of
PLAIN TRIANGLES
To the Taking of
HEIGHTS and DISTANCES,
AND TO
Plain and Mercator's SAILING.

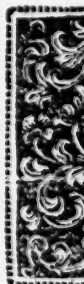
By JOHN WILSON.

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at



THE PREFACE.

READER,



*Had neither troubled
you nor my self with
a Preface, if I had
not thought what fol-
lows worth your no-
ticing.*

*In the Doctrine of the Sphere I
have only illustrated, not demonstra-
ed the Increase of the Days; he
that desires a Geometric Demonstra-
a tion*

ii The Preface.

tion of it, may read Theodosius's Spherics, Lib. 2. Pr. 19.

In the Calculation of the Problems of the Sphere, I generally (for Brevity's sake) call the Arch of the Equator, intercepted betwixt the nearest Equinoctial Point, and the Meridian passing through the Sun's Place, the right Ascension, tho' it is never so but when the Sun is in the first Quadrant of the Ecliptic; but it is easily known when it is otherwise by Probl. III.

The same way at Problem X. I call the Angle PzS the Longitude, which cannot be but when the Longitude is less than 90° ; but in all Cases that Angle known, the Longitude is easily found, if the 3d Probl. is sufficiently understood.

In

The Preface. iii

In Projection, the Problems follow so naturally from the Theorems premised, that I thought it superfluous to add any further Demonstration.

I could bring the common Excuse for appearing in Print, viz. the earnest Intreaty of Friends; but I think it weak in any Man both to cross his own Inclinations and to plague the rest of Mankind with a Tale of a new Piece, merely out of Complaisance to a Friend or Two. I'm confident the Reader will not suspect me of Vanity, when I tell him I have advanced nothing that's new.

I've here laid down the Method I, for ordinary, use in Teaching, I was obliged to print it for the Ease and

iv The Preface.

Satisfaction of such Gentlemen as were pleas'd to attend my Lessons, who found themselves much at a Loss by incorrect Copies they got of it in writ. This was the only Argument prevail'd with me to print, and 'tis the only Excuse I bring for it.

Adieu.

To the Book-Binder.

Place	Plate	at	Page
1	- - - - -		21
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Cor-

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Corrigenda.

PAge 32 Line *ult.* for AEC read ACB.

P. 40. L. 11, f. AD r. *ad.*

P. 41 L. 1, f. Ad r. *ad.*

P. 47 in Citation, f. 14.e.1. r. 14c.6.

P. 48 L. 6. f. tBC. r. ^tBC.

P. 49. L. 8. f. BM-BC r. BM-BA.

P. 52. among *data* at Case 4. put BC.

P. 89. L. 2. f. 6^h † that time, r. 6^h ‡ that time.

P. 114. L. *ult.* f. Co-Longitude, r. Long.

P. 121. L. 14. f. FC r. Fc.

P. 142. L. 11. f. ^tpam. ^tpm. r. ^spam. ^spm.

Characters

$\perp$ Perpendicular.

$\parallel$ Parallel.

R, r Radius.

S, s Sine. Σ, σ Co-Sine.

$V S$ Versed Sine.

T, t Tangent.

$\overline{7}$ or $\cot$. Co-Tangent.

AE aff. as LC . AE affected as the Angle C .

$::$ Geometrical Proportion.

$:::$ Arithmetical Proportion.

$\div$ Geometrical Proportion continued.

$\equiv$ Arithmetical Proportion continued.

$\circ$ Degrees, as 90° is 90 Degrees.

$'$ Minutes. $''$ Seconds.

$'''$ Thirds. $''''$ Fourths, &c.

$+$ More or to be added.

$-$ Less or to be subtracted.

$=$ Equal to.

$>$ Greater than. $<$ Less than.

$\times$ Multiplied into.

$\sim$ Difference when it is uncertain which is greater.

$\sqrt{\quad}$ The Square Root of.

Tri-

Trigonometry.

PART I.

Of the Construction of Tables of natural Sines, Tangents and Secants:

Definitions. Fig. I.

I.  N *Arch* is any Part of the Periphery of a Circle, as
A B.

II. A *Degree* is the 360^{th} Part of the Periphery; hence a *Semicircle* contains 180° and a *Quadrant* 90° .

III. A *Minute* is the 60^{th} Part of a Degree, and a *Second* the 60^{th} Part of a Minute; and so on, in Thirds and Fourths, &c.

N. B. Arches are the Measure of Angles, so A B is the Measure of the Angle at the Center A O B. *i. e.* If A B be an Arch of 35° . A O B is said to be an Angle of 35° . Hence, The Sine, Tangent, or Secant of any Arch, may be called the Sine, Tangent, or Secant of the Angle whose Measure that Arch is.

A

IV. The

IV. The *Chord* or Subtense of any Arch, is a right Line joining the Extremities of that Arch.

V. The *Complement* of any Arch is so much as it wants of 90° .

VI. The right *Sine* of any Arch is Half the Chord of double that Arch. So $AD = \frac{1}{2} AC$ is the right Sine of AB . Or the right Sine is a Line drawn from one Extremity of the Arch, perpendicular to a Diameter that passes through the other Extremity of that Arch, so $AD \perp HB$.

N.B. As every Chord AC subtends Two Arches AHC , ABC , the one greater, the other less than a Semicircle; so every Sine AD subtends Two Arches, AGH , AB , the one greater, the other less than a Quadrant; and these Arches AGH , AB are Complements of each other to a Semicircle.

VII. *Versed-Sine* is so much of the Diameter as is intercepted between the right Sine and the Periphery; so DB is the versed Sine of the Arch AB , and DH is the versed Sine of the Arch AGH .

VIII. The *Co-Sine* is the Sine of the Complement; so AF is the Co-Sine of AB , for it is the Sine of its Complement AG .

N.B. In

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N. B. In the Parallelogram DO the Side $DO = AF$ the Co-Sine of AB . Hence,

The Co-Sine of any Arch is so much of the Radius as lies betwixt the Center and the right Sine of that Arch.

IX. The *Whole-Sine*, or Sine of 90° is the Radius of the Circle; so GO is the Sine of the Quadrant GB .

X. The *Tangent* is a Line touching the Circle at one Extremity of the Arch, and continued till it meet another Line passing through the Center and other Extremity of that Arch; so BE is the Tangent of AB .

XI. The *Secant* is that right Line OE , which limits the Tangent BE .

XII. *Co-Tangent* and *Co-Secant* are the Tangent and Secant of the Complement, as GL , OL .

XIII. *Tables* of natural Sines, Tangents and Secants are numeral Tables, expressing the natural Sines, Tangents and Secants of the several Arches, from one Minute to 90° in Parts; of which Parts the Radius contains a certain Number taken at Pleasure, suppose 10000000; so if the Radius GO be supposed

posed to be divided into 10000000 equal Parts, AD the Sine of $AB (= 35^\circ)$ shall contain 5735764 of these Parts.

Proposition I. Fig. 2.

Given DE the Sine of any Arch DC , sought AD its Co-sine.

^a 47. e. 1. In the right angled Triangle
^b Def. 8. DEB ; $BDq - DEq^a = BEq^b = ADq$. *Q. E. F.*

Cor. Hence, $BC - BE = EC = {}^v s DC$.

Prop. II. Fig. 2.

Given the Sine of any Arch, to find the Sine of Half or Double that Arch.

1st. Given $DE = {}^s DC$, sought $DF =$

^a 47. e. 1. $S \frac{1}{2} DC$. $\frac{1}{2} V DEq + ECq^a = \frac{1}{2} V DCq$
^b 3. c. 3. $= \frac{1}{2} DC^b = DF$. *Q. E. F.*

2^d. Given DF , sought DE . The Triangles DEC , BFC are simular, for $L^ic C$
^e d. 6. is common, and DEC , BFC^c are
^d 4. e. 6. right; therefore, $BC.BF^d :: DC$.
 DE , that is, $R. \sigma :: 2s DE$. *Q. E. F.*

Prop.

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Prop. III. Fig. 2.

GIVEN KI , DE the Sines of 2 Arches KD , DC ; sought KM , the Sine of their Sum KC .

Compleat the Parallelogram $PINM$, the Triangles BDE , BIN , BOM , OIK , OIP , KIP are all simular; therefore $BD. DE^2 :: BI. IN = PM.$ and $BD. BE^2 :: KI. KP.$ and $KP \div PM = KM.$ *Q. E. F.* ^a 4. e. 6.

Prop. IV. Fig. 2.

GIVEN KM , DE the Sines of 2 Arches, sought KI the Sine of their Difference.

$BE. DE^2 :: BM. OM$ and OM given, OK is known; so $BD. BE^2 :: OK. KI.$

N. B. The Sines of lesser Arches are (at least Physically) as their Arches; for in the Beginning of a Quadrant, the Sine and Arch^b being both perpendicular to the Diameter, they must coincide, and consequently their Ratio's are equal. ^a d. 6.

Prop. V. Fig. 3.

THE Difference of the Sines of 2 Arches equally distant from 60° is

is equal to the Sine of the common Distance.

Make DE , CF the Sines of 2 Arches AD , AC , which 2 Arches are equally distant from $60^\circ = AB$. $DB = BC$ is the common Distance, whose ^{e 31. e. 1.} Sine is $DI = IC$. Draw CG ^e FE , so is $DG = DE - CF =$ the Difference of the Sines DE , CF . I say that $DG = DI$. Draw CM ,

The $\triangle^{les} MID$, MIC have the L^{les} at I ^f right, $ID = IC$ and MI common, ^{f Def. 6.} therefore $IMC^a = IMD^b =$
^{a 4. e. 1.} $BKN^c = 30^\circ$. Therefore $CMD =$
^{b 29. e. 1.} 60° ^d $MDI = MCI$, therefore
^{c Hyp.} $MD^e = DC$. But $DG = \frac{1}{2}MD$;
^{d 3. Cor.} for in the $\triangle^{les} CGD$, CGM the
^{32. e. 1.} L^{les} at G are right, the $L^{le} GDC =$
^{e Cor. 6.} GMC and GC is common, there-
^{e. 1.} fore $DG^f = GM = \frac{1}{2}DM = \frac{1}{2}DC =$
 DI . *Q. E. D.*

Cor. Hence, if the Sines of Arches to 60° are known, the rest to 90° are found by Addition only; for $'43^\circ + '17^\circ =$
 $'60^\circ + '17^\circ = '77^\circ$.

Prop.

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Prop. VI. Fig. 3.

IF 2 Arches are equally distant from 30° , the Square of the Sine of their common Distance is $\frac{1}{3}$ of the Square of the Difference of the Sines of these 2 Arches.

Make the Arches ND , NC equally distant from $NB = 30^\circ$ whose Sines are LD , PC , then is GC the Difference of the Sines; I say $DIq = \frac{1}{3} GCq$. For seeing $DI = DG$, in the right angled Triangle CGD we have DIq (i.e. DGq) $+ GCq^2 = DCq^2 =$ ^{a 47. c. 1.} _{b Cor.} $4 DIq$, therefore $GCq^2 =$ ^{a 47. c. 1.} _{b Cor.} $4 DIq$ and $\frac{GCq^2}{3} = DIq$. *Q. E. D.* ^{c ax. 3.}

Cor. Hence, if the Sines to 30° are known, the rest to 60° are easily found, for $13^\circ + V_3 \times 17^\circ q = 47^\circ$.

Prop. VII.

IF any Quantity $aec (=60)$ be divided by Two Quantities, $ac = 10$ and $ec = 15$, the Divisors and Quotes shall be reciprocally proportional.

$$\left\{ \begin{matrix} ac \\ ec \end{matrix} \right\} aec \left(\begin{matrix} e \\ a \end{matrix} \right)_{ac}$$

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we shall at the Twelfth Division find that the Sines are as their Arches, *i. e.* the last Sine is just $\frac{1}{2}$ the preceeding Sine as the last Arch is $\frac{1}{2}$ the preceeding Arch.

Now dividing the Arch of 30° into Parts, whereof the Arch $52'' . 44''' . 3'''' . 45'''''$ is one, the Quote or Number of such Parts contain'd in 30° is 2048.

But the same Arch of 30° divided into Minutes gives 1800', wherefore $1800' : 2048^a :: 52'' . 44''' . 3'''' . 45''''' : 1' : ^b$ Sine $52'' . 44''' . 3'''' . 45''''' : \text{Sine } 1' .$ ^a 7. ^b N. B. 4.

The Sine of one Minute being given, that of $2'$ is known by 2^d , that of $3'$ by 3^d ; as also that of 4, 5, 6, &c. to 30° , thence to 60° by 6^{th} , and thence to 90° by 5^{th} .

The Table of *Sines* being made, the Tables of *Tangents* and *Secants* are easily constructed by the Two following Propositions.

Prop. X. Fig. 4.

THE Co-Sine of any Arch AB is to its Sine as the Radius to the Tangent B

^a 4. e. 6.

gent of that Arch. 'Tis plain,
 $DC \cdot CB^a :: DA \cdot AF$.

Prop. XI. Fig. 4.

THE Radius is a Mean proportional
 betwixt the Co-Sine and Secant
 of any Arch; for $DC \cdot DA :: DB \cdot DF$.
Q. E. D.

Trigonometry.

PART II.

Of the Construction of Tables of Logarithms.

- I. **L**ogarithms are Numbers in Arithmetical Proportion, answering to other Numbers Geometrically proportional. So 0, 1, 2, 3, 4, &c. are the

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the Logarithms of the Geometrical Proportionals, 1, 10, 100, 1000, &c. But that the Logarithms of intermediate Numbers

Propor. Geo.
1
10
100
1000
10000
100000
1000000

Propor. Arith.
0.0000000
1.0000000
2.0000000
3.0000000
4.0000000
5.0000000
6.0000000

(2, 3, 4, &c.) may be found, we shall affix to the several Logarithms a certain Number of Cyphers (suppose Seven) then the Logarithm of 1 is 0.0000000, that of 10 is 1.0000000, that of 100 is 2.0000000, &c. And then the Logarithms first placed (*viz.* 0, 1, 2, &c.) may be called the Characteristicks of the several Logarithms. So 2 is the Characteristick of all the Logarithms betwixt that of 100 and that of 1000.

II. Four Numbers are said to be in Arithmetical Proportion, when the Difference of the 1st and 2^d is equal to that of the 3^d and 4th.

So 3.5 :: 16.18. and 7.4 :: 15.12.

III. Three Numbers are in continued Arithmetical Proportion, when the Difference of the 1st and 2^d is equal to that of the 2^d and 3^d.

B 2

So

So $3.5.7 \equiv$ and $8.5.2 \equiv$ and $0.$
 $1.2 \equiv$

Prop. I.

IF 4 Numbers be in Arithmetical Proportion, the Sum of the Extremes is equal to that of the middle Terms. If $9.11::12.14$ then is $9 + 14 = 11 + 12 = 23$.

Prop. II.

IF 3 Numbers are in Arithmetical Proportion, the Sum of the Extremes is double the middle Term. If $8.13.18 \equiv$ then is $8 + 18 = 2 \times 13 = 26$.

Prop. III.

THe Sum of the Logarithms of 2 entire Numbers is equal to the Logarithms of their Product, when the Logarithm of Unity is 0. Make the Numbers 100 and 1000, whose Logarithms are 2 and 3, let the Logarithm of

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of their Product (100000) be x . then

$$1. 100^a : : 1000. 100000.$$

and $0. 2^b : : 3. x$. There-
fore, $0 + x^c = 2 + 3 = 5 = x$. ^a def. mult.
^b def. 2.
^c 1.

Q. E. D.

Cor. Hence, Addition of Logarithms is the same as Multiplication of their correspondent Numbers.

Prop. IV.

THE Difference of the Logarithms of Two entire Numbers is equal to the Logarithm of their Quot, when the Logarithm of Unity is 0

Make the Numbers 100 and 100000, whose Logarithms are 2 and 5, and make the Logarithm of their Quot (1000) = x . Then 1.

$$1000^a : : 100. 100000, \text{ and } 0. x^b : : 2. 5.$$
^a def. div.
^b def. 1.

Therefore $0 + 5^c = x + 2 =$
5 and transposing 2, $x = 5$
 $- 2 = 3$. *Q. E. D.* ^c 1.

Hence, Subtraction of Logarithms is the same as Division of their correspondent Numbers. *Prop.*

Prop. V.

THe Logarithm of any Number is $\frac{1}{2}$ the Logarithm of its Square, and $\frac{1}{3}$ the Logarithm of its Cube, when the Logarithm of Unity is 0.

Make the Number 100, whose Logarithm is 2, and make the Logarithm of its Square x , and of its Cube z .

$1 \cdot 100 :: 100 \cdot 10000$
 $0 \cdot 2 :: 2 \cdot x$) Then $0 + x - 2 + 2 = 4$ $(1 \cdot 100 :: 10000 \cdot 1000000$
 $0 \cdot 2 :: 2 :: 4 (=x) \cdot z$
 Then $0 + z - 2 + 4 = 6 = 3x$.

Prop. VI.

TO find a Mean Proportional Geometrical betwixt Two Numbers given. The Square Root of their Product is the Mean, by 20 of *Euclid's* 7th Book.

Prop. VII.

TO find a Mean Proportional Arithmetical between 2 Numbers given.

Prop. 2.

sought.

Half their Sum^a is the Mean

Prop.

Prop. VIII.

TO find the
 Logarithm
 of any Number.
 Make the Num-
 ber 9, because it
 is between 1 (A)
 and 10 (B) find C
 a Mean propor-
 tional Geome-
 trical betwixt
 A and B (aug-
 menting each
 by as many Cy-
 phers as their
 Logarithms con-
 tain) and, be-
 cause C is less
 than 9.0000000,
 find D a Mean
 proportional be-
 tween B and C,
 and between B
 and D, another
 Mean E; and so
 on

<i>Proportions.</i>		<i>Logarit.</i>
A	1.0000000	0.0000000
C	3.1622777	0.5000000
B	10.0000000	1.0000000
B	10.0000000	1.0000000
D	5.6234132	0.7500000
C	3.1622777	0.5000000
B	10.0000000	1.0000000
E	7.4989421	0.8750000
D	5.6234132	0.7500000
B	10.0000000	1.0000000
F	8.6596432	0.9375000
E	7.4989421	0.8750000
B	10.0000000	1.0000000
G	9.3057204	0.9687500
F	8.6596432	0.9375000
G	9.3057204	0.9687500
H	8.9768713	0.9531250
F	8.6596432	0.9375000
G	9.3057204	0.9687500
I	9.1398170	0.9609375
H	8.9768713	0.9531250
I	9.1398170	0.9609375
K	9.0579777	0.9570312
H	8.9768713	0.9531250
K	9.0579777	0.9570812
L	9.0173333	0.9550781
H	8.9768713	0.9531250

Proportions.		Logarit.
L	9.0173333	0.9550781
M	8.9970796	0.9541015
H	8.9768713	0.9531250
L	9.0173333	0.9550781
N	9.0072008	0.9545898
M	8.9970796	0.9541015
N	9.0072008	0.9545898
O	9.0021388	0.9543457
M	8.9970796	0.9541015
O	9.0021388	0.9543457
P	8.9996088	0.9542236
M	8.9970796	0.9541015
O	9.0021388	0.9543457
Q	9.0008737	0.9542846
P	8.9996088	0.9542236
Q	9.0008737	0.9542846
R	9.0002412	0.9542541
P	8.9996088	0.9542236
R	9.0002412	0.9542541
S	8.9999250	0.9542388
P	8.9996088	0.9542236
R	9.0002412	0.9542541
T	9.0000831	0.9542465
S	8.9999250	0.9542388
T	9.0000831	0.9542465
V	9.0000041	0.9542427
S	8.9999250	0.9542388

on, till you come to G, which is greater than 9.00000000. Then between G and F (which F is next to 9.00000000, of all these that are less than it) find another Mean H, and another I betwixt G and H; betwixt which I (because it is greater than 9.00000000) and H find a Mean K, and so on till at the 26th time you find 9.00000000 (*i. e.* 9) whose Logarithm is easily found. For as we found a Geometrick Mean betwixt A and B, so

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so an Arithmetick Mean betwixt their Logarithms will be the Logarithm of c , and so at last the Logarithm of 9.0000000 will be found to be 0.9542425.

In the same Manner are found the Logarithms of Numbers betwixt 1 and 9, betwixt 10 and 100, &c. But the Logarithms of compound Numbers are more easily found by Addition by *Prop. III.*

Proportions.		Logarit.
V	9.0000041	0.9542427
X	8.9999650	0.9542428
S	8.9999250	0.9542188
—		—
V	9.0000041	0.9542427
Y	8.9999845	0.9542421
X	8.9999650	0.9542428
—		—
V	9.0000041	0.9542427
Z	8.9999943	0.9542422
Y	8.9999845	0.9542421
—		—
V	9.0000041	0.9542427
&	8.9999992	0.9542424
Z	8.9999943	0.9542422
—		—
V	9.0000041	0.9542427
A	9.0000016	0.9542425
&	8.9999992	0.9542424
—		—
A	9.0000016	0.9542425
B	9.0000004	0.9542425
&	8.9999992	0.9542424
—		—
B	9.0000004	0.9542425
C	8.9999998	0.9542425
&	8.9999992	0.9542424
—		—
B	9.0000004	0.9542425
D	9.0000000	0.9542425
C	8.9999998	0.9542425

C

Prop.

Prop. IX.

TO find the Logarithm of an entire Number that is greater than 10000.

Make the Number 3567894, cut off from the right Hand so many Figures, as what Remains on the left may be found in the Tables, as here 894.

To the Logarithm of 3567 add that of 1000, so you have 6.552303 for the Logarithm of 3567000, which being yet less than the Logarithm of 3567894, find 122 the Difference betwixt the Logarithm of 3568 and that of 3567; then say, If 100 give 122, what will 894 give? The Answer 109 added to the Logarithm of 3567894 gives 6.552412, for the Logarithm of 3567894.

N.	Logarith.
3567	3.552303
1000	3.000000
3567000	6.552303
	+ 109
3567894	6.552412
	3568

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N.	Logarith.
3568	3.552425
3567	3.552303
100 : 122 :: 894	122

$$\begin{array}{r}
 122 \\
 \hline
 1788 \\
 1788 \\
 \hline
 894 - 100) 109068 (109.
 \end{array}$$

Cor. Hence may be constructed the Tables of *Logarithmick* or *Artificial* Sines, and by it the Tables of artificial Tangents and Secants.

Trigonometry.

PART III.

Of the Solution of plain Triangles.

Prop. I. Fig. 5, 6, 7.

IN any plain Triangle ABC, the Sides are proportional to the Sines of the opposite Angles.

C 2

About

About the $\triangle^{lc} ABC$ describe
^a 5. e. 4. a ^a Circle, from the Center ^d D
^b 12. e. 1. to the several Sides ^b draw Per-
^c 3. e. 3. pendiculars, which shall bi-
^d 30. e. 3. sect the Sides ^c at K, I, H, and
the Arches ^d at G, E, F, from
the Center to the several An-
gles draw right Lines DA, DB,
DC. $AK^c = \frac{1}{2} AB$ is the
^e def. 6. ^c Sine of $L^{lc} ADG = \frac{1}{2} L^{lc} ADB$
^f 20. e. 3. $=^f ACB$. $AH = \frac{1}{2} AC$ is
the Sine of $L^{lc} ADE^d = \frac{1}{2}$
 $ADC^f = L^{lc} ABC$. And
^h 15. e. 5. it is plain, that $AB.AC^h::$
 $\frac{1}{2} AB. \frac{1}{2} AC:: AK.AH::^s ACB.$
 $^s ABC. Q. E. D.$

Prop. II. Fig. 8, 9, 10. Plate 2.

IN any right angled $\triangle^{lc} ABC$ right
angled at B, making any Side the
Radius, the other Two Sides will be
Sines, Tangents and Secants.

^a def. 10. 1. Make CB the Radius,
then is AB the ^a Tangent, and
^b def. 11. AC the ^b Secant of the Angle C .

2. If

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2. If AB be the Radius, then is CB the ^aTangent, and AC the Secant of the Angle A .

3. If CA be the Radius, then is AB the ^cright Sine, ^c def. 6. and CB the ^dCo-Sine of the ^d N.B.d.8. Angle C , consequently CB is the right Sine of the Angle A .

Prop. III. Fig. II.

IN any Triangle ACB , as the Sum of the Sides is to the Difference of the Sides, so is the Tangent of $\frac{1}{2}$ the Sum of the opposite Angles to the Tangent of $\frac{1}{2}$ their Difference, $CB + BA$.

$$CB - BA :: T \frac{1}{2} A + A CB . T \frac{1}{2} A - A CB .$$

Produce AB to H making $BH = BC$; cut off $BI = AB$, so is AH the Sum and IH the Difference of the Sides.

Join CH , draw $BE^a \perp CH$, ^a 12. e. 1. that BE shall bisect ^b CH , and ^b Cor. 3. e. 3. ^c the Angle CBH , so is CBE ^c 8. e. 1.

$= \frac{1}{2} CBH^d = \frac{1}{2}$ the Sum of the ^d 32. e. 1. Angles $A + ACB$, draw ^e BD ^e 31. e. 1. and IG each $\parallel AC$, then is

DBH

‡ 29. e. I.

§ Constr.

‡ 5. e. I.

‡ 4. e. I.

$DBH^f = A$ the greater Angle, and $CBD^f = ACB$ the lesser Angle: Take $HF = CD$ join BF , in the $\Delta^{les} CBD$, HBF ; $CB^g = BH$, $CD^g = HF$, and the $L^{ic} BCD^k = BHF$; therefore $HBF^l = CBD^f = ACB$, consequently $DBF = DBH - HBF^f = A - ACB$ the Difference of the Angles: And, 'cause $BD^l = BF$, BE bisects DF , and the Angle DBF ; therefore the $L^{ic} DBE = \frac{1}{2}$ the Difference of the Angles $= \frac{1}{2} A - ACB$.

‡ 2. e. 6.

‡ ax. 3.

Because $IG^g \parallel DB \parallel AC$, and $AB^g = BI$; therefore $CD^m = DG^g = HF$, and, taking away FG , which is common, $DF = GH$, and consequently $\frac{1}{2}GH = \frac{1}{2}DF = DE$.

‡ def. 10.

On B as a Center with the Radius BE describe a Circle, then is EC the o Tangent of the $L^{ic} CBE^d = \frac{1}{2}$ the Sum, and $ED =$ the o Tangent of the $L^{ic} DBE$

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$\angle^{te} DBE = \frac{1}{2}$ the Difference
of the Angles. And $HA \cdot HI^q$
 $:: HC \cdot HG^r :: \frac{1}{2} HC : \frac{1}{2} HG ::$
 $EC \cdot ED. Q. E. D.$

^q 4. e. 6.
^r 15. e. 5.

Prop. IV. Fig. 12.

IN any plain Triangle ADE , the Base
is to the Sum of the Sides as the
Difference of the Sides to the Difference
of the Segments of the Base,
viz.

$AE \cdot AD + DE :: AD - DE \cdot AB -$
 $BE. i. e. AE \cdot AC :: AP \cdot AI.$

Draw $DB^a \perp AE$; upon D
as a Center with the Radius
 DE the Lesser Side describe a Circle
cutting the Base AE at I , and the Side
 AD at P , produce AD till it meet the
Periphery at C .

^a 12. e. 1.

Because $DC^b = DE = DP$,
and $BI^c = BE$, 'tis plain AC
shall be the Sum of the Sides
 AP their Difference and AI
the Difference of the Seg-
ments of the Base. But

^b def. 15.

^c 1.

^c 3. e. 3.

$AEXAI$

^d 37. e. 3.
^c 16. e. 6.

$A E \times A I^d = A C \times A P$, where-
fore $A E . A C ::^c A P . A I$. Q.
E. D.

Fig. 13, 14,
15.

In the following Proportions for the Solution of plain Triangles; for right angled Triangles take the Triangle $A B C$, whose right Angle is B , and for oblique angled Triangles, take the Two Triangles $D E F$, $I G K$ and let $I G K$ be divided into Two right angled Triangles by the Perpendicular $I L$.

Solution of right angled Triangles.

	Given.	Sought.	Proportions.
a 1	1. $A C . A B$	$A C B$	$A C . R^a :: A B .^s C$
b 2	2. $L^c A . L^c C . A B$	$B C$	$R . A B^b ::^t A . B C$
c 3	3. $B A . B C$	$L^c A$	$B A . R^b :: B C .^t A$
d 4	4. $L^c A . L^c C . A C$	$C B$	$R . A C^a ::^s A . B C$

Solution of Oblique angled Triangles.

	Given.	Sought.	Proportions.
5	$L^c D . L^c E . D F$	$D E$	$^s E . D F^a ::^s F . D E$
6	$D E . E F . L^c F$	$L^c D$	$D E .^s F^a :: E F .^s D$
7	$D E . E F . L^c E$	$L^c D . F$	$D E . E F . D E - E F^c :: T_{\frac{1}{2}} D . F$
8	$G I . I K . G K$	$L^c G I K$	$T_{\frac{1}{2}} D . F$ $G K . I K . I G^d :: I K - I G$ $. K L - L G$ $I G . R :: G L .^s G I L, &c.$

N. B.

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N. B. 1. From all the Angles given^c the Ratio's of the Sides may be found, but not the Sides themselves. ^{c 1.}

2. In Case 6th it must be known if the Angle sought D be greater or less than a right Angle, for^f every Sine has two Angles (the one greater the other less than a Quadrant) and only one Angle being given, that sought may be either greater or less than a right Angle, and yet have the same Sine. ^{f N. B. def. 6.}

3. One Angle given, the Sum of the other Two is its Complement to 180° .

4. $\frac{1}{2}$ the Sum $+$ $\frac{1}{2}$ the Difference of any Two Quantities is $=$ greater and $\frac{1}{2}$ the Sum $- \frac{1}{2}$ the Difference of any Two Quantities is $=$ lesser.

Make a the greater, b the lesser Quantity.

$$\text{Add } \begin{cases} \frac{1}{2} \text{ Sum} = \frac{1}{2} a + \frac{1}{2} b \\ \frac{1}{2} \text{ Diff.} = \frac{1}{2} a - \frac{1}{2} b \end{cases}$$

$$\text{Substr. } \begin{cases} \frac{1}{2} \text{ Sum} = \frac{1}{2} a + \frac{1}{2} b \\ \frac{1}{2} \text{ Diff.} = \frac{1}{2} a - \frac{1}{2} b \end{cases}$$

$$\frac{1}{2} \text{ Sum} + \frac{1}{2} \text{ Diff.} = a = \text{the greater.}$$

$$\frac{1}{2} \text{ Sum} - \frac{1}{2} \text{ Diff.} = b = \text{the lesser.}$$

D

Trigo

Trigonometry.

PART IV.

Of the Solution of Spherique Triangles.

Definitions.

I. **A** Globe or Sphere is a solid Body, conceived to be generated by the Rotation of a Semicircle AEB about its Diameter AB .

II. The Center of the Sphere is the same with that of the generating Semicircle D .

Cor. Hence, all the right Lines drawn from the Center to the Surface of the Sphere are equal as $DE = DC = DL = DF$.

III. The Axis of the Sphere is the Diameter of the generating Semicircle AB .

IV. The

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IV. The Poles of the Sphere are the Extremities of its Axis A. B.

V. A great Circle on the Sphere is whose Plan passes through the Center of the Sphere. Hence,

Cor. 1. All great Circles have the same common Center, *viz.* that of the Sphere.

Cor. 2. Every great Circle divides the Sphere into Two equal Segments or Hemi-Spheres.

VI. The Axis of a great Circle is a right Line drawn through its Center perpendicular to its Plan, and terminated by the Surface of the Sphere on both Sides.

VII. The Poles of any Circle are the Extremities of its Axis.

VIII. A Spherique Triangle is contained under the Arches of Three great Circles, as A E C.

IX. A Spheric Angle is the Inclination of the Peripheries of Two great Circles, and is the same with the Inclination of the Plans of these Circles, as $L^{\text{ic}} E A C = L^{\text{ic}} E D C$.

D 2

X. Lesser

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X. Lesser Circles are, whose Plans do not pass through the Center of the Sphere: They have the same Poles and Axis with these of the great Circles to which they are parallel.

Prop. I. Fig. 16.

Great Circles AEB , ACB bisect each other,

For their common Section
^a 3. c. 11. ^a is a right Line AB passing
^b Cor. def. 5. through D^b the Center of both
 Circles, and consequently 'tis
 a Diameter to both. Q. E. D.

Prop. II. Fig. 16.

A Great Circle AEB is 90° distant from its Pole C .

Because $AD^a = DC$ and
^a C. def. 2. $DC^b \perp AD$, therefore $AC =$
^b def. 6. 90° and consequently CB^c
^c I. $= 90^\circ$. The same way CE ,
 CF are Quadrants. Q. E. D.

Cor. The

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Cor. The Distance of any lesser Circle from its Poles $= 90^\circ \pm$ its Distance from its parallel great Circle.

Prop. III. Fig. 16.

IF a great Circle ECF be describ'd from the Pole A , the Arch EC , lying betwixt AC and EA , which Form the Angle at A , is the Measure of that Angle EAC .

For $AC^a = AE = 90^\circ$, therefore the Angle $ADC^b = 90^\circ$
 $b = ADE$, wherefore $CDE^b =$
 $EC^c =$ to the Inclination of
the Two Plans ACD , AED^d
 $= L^e EAC$. *Q. E. D.*

^a 2.
^b N. B. D.
3. P. I.
^c d. 6. e. 11.
^d d. 9.

Prop. IV. Fig. 16.

A Great Circle ACB cutting another $AEBF$ makes the Angles *Deinceps* EAC ,
 $CAF^a =$ to the Arches EC ,
 $CF^b =$ to a Semicircle or 180°
 $c =$ to Two right Angles.

^a 3.
^b I.
^c N. B. def.
3. P. I.

Prop.

Prop. V. Fig. 16.

^a 3.
^b N. B. d.
 3. P. I.
^c 15. c. II.

THE Angles at the Vertex EAC, FAL are equal, for $L^c EAC^a = EC^b = EDC^c = FDL^b = LF^a = FAL$. Q. E. D.

Prop. VI.

ANGLES EAC, EBC at a Semicircle's Distance, are equal.

^a 3. For $EC^b = EDC$ the Inclination of the Two Plans^a, is the Measure of both Angles.

Prop. VII.

TWO Triangles, having one Angle and the Sides forming that Angle equal, are every way equal.

Prop.

Prop. VIII.

TWO Triangles, having one Side and the adjacent Angles equal, are every way equal.

Prop. IX.

EQUILATERAL Triangles are likewise equiangular.

Prop. X.

ISOSCELES Triangles have the Angles, both above, and below the Base, equal.

Prop. XI.

IF the Angles at the Base are equal, the Triangle is an Isosceles.

In this and the Four preceeding, the Demonstration is the same as in plain Triangles.

Prop.

Prop. XII.

ANy Two Sides of a Triangle taken together are greater than the Third.

For as in a plain Surface, a right Line is the nearest Distance betwixt Two Points, so in a Spherique Surface, the Arch of a great Circle.

Prop. XIII. Fig. 16.

ANy Side of a Triangle AEC is less than a Semicircle.

Produce AE , AC till they meet at B , then are AEB , ACB Semicircles, and 'tis plain $AEB > AE$ and $ACB > AC$; also $ECF > EC$. *Q. E. D.*

Prop. XIV. Fig. 16.

ALL the Sides of a Triangle are less than a Circle.

For $EC^a < EB + BC$, therefore $AE + AC + EC < AEB + ACB$. *i. e.* $< a$ Circle.

Q. E. D.

Prop.

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Prop. XV. Fig. 16.

IN any Triangle AEC , the greater Side subtends the greater Angle AEC .

Because $AEC^a > EAC$, ^{a Hyp.}
make $AEd = EAC$, then is ^{b 11.}
 $Ed^b = Ad$ and $AC = Ad +$ ^{c 12.}
 $dC = Ed + dC^c > EC$. *Q. E. D.*

Prop. XVI. Fig. 16.

IN any Triangle AEC , if the Sum of the Sides $AE + EC$ be $>$, $=$ or $<$ than a Semicircle, the internal Angle EAC , at the Base AC , will be $>$, $=$ or $<$ than the external and opposite Angle ECB , and therefore the Sum of the Angles $A + ACE$ $>$, $=$ or $<$ than Two rights.

If $AE + EC >$, $=$ or $< AEB$,
then is $EC >$, $=$ or $< EB$ ^{b 6.}
and $EC^b = A^c >^d =$ or $<$ ^{c 15.}
 ECB , and consequently $A +$ ^{d 10.}
 $ACE >$, $=$ or $< ECB + ACE$ ^{e 4.}
which are ^e equal to 2 rights.

E

Prop.

Prop. XVII. Fig. 17.

IN any Triangle ABC whose Sides are AB , AC , BC , and their Poles E , F , D , the Triangle EDF is the Complement of the Triangle ABC ; that is,
 1. The Sides of the $\Delta^{lc} EDF$ are the Complements of the Angles of ABC to Semicircles; And,

2. The Angles of EDF are the Complements of the Sides of the Triangle ABC to Semicircles.

Upon A , B , C as Poles describe the great Circles $FIEL$, TED , $VESD$,
 then is $AE^a = 90^\circ = BE$, therefore E^a is the Pole of AB ; also
 $BD^b = 90^\circ = CD$, therefore D^b is the Pole of BC . Again, $AF^c = 90^\circ = CF$, therefore F^c is the Pole of AC .

^b ax. 2.

^c 1.

^d 3.

1. $FG^d = 90^\circ = EL$, then

$FE^b = GL^c =$ complement

IG i. e. L^lc to 180°

$TE = 90^\circ = DN$, then $DE^b = TN^f =$
 Compl. RN i. e. L^lc to 180°

$FV = 90^\circ = DO$, then $DF^b = VO^c =$
 Compl. OS i. e. L^lc to 180°

2. $L^lc E$

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$$2. L^c E^d = IR = AI + BR - AB = 2 \text{ Quadrants} - AB$$

$$L^c D^d = ON = BN + CO^d 3. - BC = 2 \text{ Quadrants} - BC$$

$$L^c F^d = SG = GA + CS - AC = 2 \text{ Quadrants} - AC, Q. E. D.$$

Prop. XVIII.

Equiangular Triangles are likewise equilateral.

For their Complements^a are equilateral, and therefore^b equiangular, wherefore the Triangles themselves are equilateral. *Q. E. D.*

Prop. XIX. Fig. 17.

THE Three Angles of any Spherique Triangle are $>$ than 2 and $<$ than 6 Rights. For 6 Rights

$$-A - B - C^a = DF + FE + DE^b 17. 14.$$

$< 4 R$; and transposing $-A - B - C$ we have $6 R < 4 R + A + B + C$, and taking $4 R$ from both Sides, we have $2 R < A + B + C. Q. E. D.$

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The 2d Part is plain; for the external and internal Angles together are equal to 6 right Angles. *Ergo, &c.*

Prop. XX. Fig. 16.

A Circle $FCE L$ passing through L , C the Poles of another great Circle $A F B E$ makes with it right Angles.

For it passes thro' ^a $L C$, which is ^b right to the Plan $A F B E$, therefore $FCE L$ ^c is perpendicular to that Plan.

Q. E. D.

Prop. XXI. Fig. 16.

IF a great Circle $FCE L$ be right to another $A F B E$, it shall pass thro' its Poles C, L .

Because its Plan passes ^a thro' D , and is right ^b to the Plan $A F B E$; CD , which is ^c right to the same Plan, will be ^d in its Plan. *Ergo, &c.*

Prop.

Prop. XXII. Fig. 18.

IF to a great Circle $AFBE$, from a Point R , which is not its Pole, there be drawn Arches of great Circles RA , RB , RT , RV , that Arch RA , which passes thro' c the Pole of $AFBE$, is the greatest; its Complement to a Semicircle RB is the least; and any Arch RT , which is nearer to RA , is greater than that RV which is farther removed from it, and the Arches shall make obtuse Angles towards RA .

$BRCA^a \perp AFB$. Draw RS ^{a 20.}
 $\perp$ to the Plan AFB , that RS ^{b 38. e. II.}
^b shall fall upon AB^c the com- ^{c I.}
 mon Section and common
 Diameter of the Circles ACB , AFB ,
 join the Right Lines RA , RB , RT ,
 RV .

In the $\Delta^{les} RSA$, RST , RSV , ^{d D. 3. e. II.}
 RSB , ^d right angled at s ; RA_q ^{e 47. e. I.}
^c $= RS_q + SA_q^f > RS_q + ST_q^e$ ^{f 7. e. 3.}
^{g 28. e. 3.}
 i. e. RT_q , therefore $RA > RT$
 and the Arch $RA^g > RT$; the same way
 RT_q

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$RT_q^c = RS_q + ST_q^f > RS_q + SV_q^i.e.$
 $RV_q^f > RS_q + SB_q^i.e. RB_q.$ Ergo, &c.

2. Because c is the Pole of the great Circle $AFBE$, therefore CTA^a shall be a right Angle, and consequently RTA shall be obtuse. Q.E.D.

Prop. XXIII. Fig. 18.

IN any Spherique Triangle right angled at A , the Sides containing the right Angle, are of the same Affection with their opposite Angles.

For if $AC = 90^\circ$ then G is the Pole of the Circle $AFBE$ and CTA^b a right Angle $= 90^\circ^b = CVA$. If $AC > 90^\circ$ then $are RTA, RVA$ obtuse Angles. But if $AC < 90^\circ$, then are RTA, RVA , less than CTA, CVA^b i.e. less than right Angles. Q.E.D.

N.B. Two Arches or Angles are of the same Affection, when they're at once $>, =$ or $< 90^\circ$.

Prop. XXIV.

Prop. XXIV. Fig. 18.

IF the Two Sides or Angles of a right angled Triangle be of the same Affection, the Hypothenuſe will be leſs than 90°.

In the Triangle BRV, be-
 cauſe $BR^a < 90^\circ$, i. e. BC and ^{a Hyp.}
 $BV^a < 90^\circ$, i. e. BF, then is ^{b 22.}
 $RV^b < RF$, which $RF^c = a$ ^{c 2.}
 Quadrant.

2. Becauſe $AR > AC$, i. e. 90°, and
 $AV > AF$ i. e. 90°, therefore $RV^b <$
 RE i. e. 90°. Q. E. D.

Prop. XXV. Fig. 18.

IF the Two Sides of a right angled Triangle be of a different Affection, the Hypothenuſe will be greater than a Quadrant.

If $AR > AC$ i. e. 90° and
 $AT < AF$ i. e. 90°, then is ^{b 22.}
 $RT^b > RF^c$ i. e. 90°. Q. E. D. ^{c 2.}

Prop. XXVI.

Prop. XXVI. Fig. 18.

IF the Hypothenuſe be $>$ or $<$ than 90° , the Sides and ^aconſequently their oppoſite Angles will accordingly be of the ſame or different Affections. It follows from the Two preceeding Propoſitions.

Prop. XXVII. Fig. 19.

IN any Triangle ABC , if the Angles B, C , at the Baſe be of the ſame Affection, the Perpendicular AD ſhall fall within the Triangle.

2. But if the Angles b, c , be of different Affections, the Perpendicular AD ſhall fall without the Triangle.

1. In the Firſt Caſe, if AD be not the Perpendicular, let AE without the $\triangle$ be it, then in the right angled $\triangle^{les} AEC$,
²³ AE^2 Aff. as $L^c C$ and
 the ſame AE^2 Aff. as ABE ,
 wherefore ABE Aff. as C contra Hyp.

2. In

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2. In the $\triangle^{lc}abc$, if ad be not the Perpendicular, let ae within the $\triangle^{lc}$ be it. Then in the right angled Triangles aeb , aec , ae^a affect. as b , and ae^a affect. as c : wherefore b and c are of the same affection *contra hyp.* Q. E. D.

Prop. XXVIII. Fig. 20. Plate 3.

IN the Triangles BAC , BHE right angled at A and H , and having the same acute Angle B , the Sines of the Hypothenufes BE , BC are as the Sines of the Perpendiculars HE , CA .

BT is the common Section of the Plans of the Circles $BC T$, BHT , then ER , CP the Sines of BE , BC , ^aare parallel; EF , CD the Sines of HE , AC being ^bperpendicular to the Plan BHT , are likewise ^cparallel, therefore the Plans REF , PCD ^dare parallel, and RF ^e $\parallel PD$, and consequently the Triangles REF , PCD ^fare similar, and $RE.CP^g::EF.CD$. Q. E. D.

^a D. 6. P. I.
& 28. e. I.
^b D. 4. e. II.
^c 6. e. II.
^d 15. e. II.
^e 16. e. II.
^f 10. e. II.
^g 4. e. 6.

F

Prop. XXIX.

Prop. XXIX. Fig. 20.

IN the same Triangles, the Sines of the Bases BH , BA are as the Tangents of the Perpendiculars HE , AC .

HK , AQ the Sines of BH , BA are ^aparallel, and HG , AI the Tangents of HE , AC being ^bperpendicular to the Plan BHT , are likewise ^cparallel, therefore the Plan KHG is ^d $\parallel QAI$ and $KG^e \parallel QI$, therefore the $\Delta^{les} KHG$, QAI^f are similar, and $KH : QA^g :: HG : AI$. *Q. E. D.*

^a D. 6. P. 1.

& 28. c. 1.

^b D. 10. P.

1. and D. 4.

^c 11.

^d 6. c. 11.

^e 15. c. 11.

^f 16. c. 11.

^g 10. c. 11.

^h 4. c. 6.

Prop. XXX. Fig. 21.

IN any Triangle ABC right angled at A , $\sigma L^c B$ at the Base is to the Sine of the vertical Angle ACB , as σ Perpendicular CA to R .

Produce BA , BC , CA , so as BE , BF , CI , CH may be Quadrants; upon B and C as Poles, describe the great Circles

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cles EFG, IHG; the Angles at I, H, E, F^a shall be right, and L^{ic} A is also^b right, therefore D^c is the Pole of BAE and^c G the Pole BCFI; AE is the Complement of AB to 90°, the Angle B^d = EF = DG, for DF is the Complement of both to 90°: 'cause CD is the Complement of AC and also of DH, therefore AC = DH.

a 20.
b Hyp.
c 21.
d 3.
e 28.
f 23.

In the Triangles HIC, DFC right angled at F and I, and having the same Angle c^f acute, because BA < BE i. e. 90°, 'DF. 'HI::'CD. R. that is, σB:σBCA::σCA.R. Q. E. D.

Prop. XXXI. Fig. 21.

AS σ Base is to σ Hypothenufe, so R to σ Perpendicular.

In the Δ^{ics} EAD, FCD right angled at E and F, and having the same common Angle D^a acute, for AE < BE i. e. 90°, 'AE. 'CF^b :: R. 'CD, that is, σAB. σBC:: R. σAC. Q. E. D.

a 23.
b 28.

F 2

Prop. XXXII.

Prop. XXXII. Fig. 21.

AS the 'Base to R, so is T Perpendicular to T L^{ic} at the Base.

In the Δ^{les} BAC, BEF right angled at A and E, and having their common Angle B^a acute, for
^a23.
^b29.
 $AC < AD$ i. e. 90° , 'BA. 'BE^b
 $\therefore$ 'AC. 'EF. Q. E. D.

Prop. XXXIII. Fig. 21:

AS σ vertical Angle to R, so is T Perpendicular to T Hypothenufe.

In the Δ^{les} GIF, GHD right angled at I and H, and having their common Angle G^a acute, for $HD < HC$ i. e. 90° , 'GH.
^a23.
^b29.
 $R^b \therefore$ 'DH. 'FI. Q. E. D.

Prop. XXXIV.

Prop. XXXIV. Fig. 21.

A S Hypothenuſe to R :: Perpendicu-
lar to L^{lc} at the Baſe.

In the $\Delta^{les} G I F, G H D$ right
angled at I and H, and ha- ^a 23.
ving their common Angle G ^c 28.
acute, for $H D \angle H C$, i. e. 90° : $\angle I F. \angle G F$
:: $\angle H D. \angle G D$. Q. E. D.

Prop. XXXV. Fig. 21.

A S R. σ Hypothenuſe :: L^{lc} vertical
to L^{lc} at the Baſe.

In the $\Delta^{les} H I C, D F C$ right ^a 23.
 L^{lc} at I & F, and having their ^b 29.
common Angle c acute, for $D F \angle G F$,
i. e. 90° , $\angle I C. \angle C F$:: $\angle H I. \angle D F$. Q. E. D.

The

The 16 Cases for right angled Spherique Triangles.

	Given.	Sought.	Proportions.
a 30.	1 A C. L^c C.	B.	R. σ A C ^a :: s C. σ B. σ A C.
b 23.	2 A C. L^c B. op.	C.	τ A C. R ^a :: σ B. s C.
	3 L^c B. L^c C.	A C.	s C. R ^a :: σ B. σ A C.
c 31.	4 B A. C A. sides.	B C.	R. σ B A. C :: σ A C. σ B C.
	5 A B. B C. bas. hyp.	A C.	σ A B. R ^c :: σ B C. σ A C.
d 32.	6 A B. A C. sides.	B.	s A B. R ^d :: t A C. t B.
	7 A B. L^c B. adj.	A C.	R. s A B ^d :: t B. t A C.
	8 A C. L^c B. op.	A B.	t B. R ^d :: t A C. s A B.
e 33.	9 B C. hyp. L^c C.	A C.	R. σ C ^e :: t B C. t A C.
	10 A C. L^c C. adj.	B C.	σ C. R ^e :: t A C. t B C.
	11 B C. hyp. A C. side	C. adj.	t B C. R ^e :: t A C. σ C.
f 34.	12 B C. hyp. L^c B.	A C. op. side	R. s B C ^f :: s B. s A C.
	13 A C. L^c B. op.	B C.	s B. s A C. f :: R. s B C.
	14 B C. hyp. side A C.	B. op.	s B C. R ^f :: s A C. s B.
	15 L^c B. L^c C.	B C.	t C. R ^g :: τ B. σ B C.
g 35.	16 B C. hyp. L^c C.	B.	R. σ B C ^g :: t C. τ B.

Prop. XXXVI. Fig. 22.

IN oblique-angled Spherique Triangles B C D, $b c d$, If a Perpendicular c A from the vertical Angle c to the Base B D or $b d$, divide the Triangle into Two other Triangles B C A, D C A, or $b c a$, $d c a$ right angled at A or a , The Co-sines of the L^{ics} at the Base B and D, (b and d) are as the Sines of the vertical L^{ics}

B C A,

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BCA, DCA, bca , dca . For
 $\sigma B. 'BCA' :: \sigma AC. R^a :: \sigma D.$ ^{a 30.}
 $'DCA. Q. E. D.$

Prop. XXXVII. Fig. 22.

THE σ of the Sides BC, DC, bc , dc ,
are as the σ of the Bases
BA, DA, ba , da . For $\sigma BC.$ ^{a 31.}
 $\sigma BA' :: \sigma AC. R^a :: \sigma DC. \sigma DA. Q. E. D.$

Prop. XXXVIII. Fig. 22.

THE Sines of the Bases BA, DA, ba ,
 da , are reciprocally as the r of
 $L^{ics} B, D, b, d$, at the Base BD, bd .

Because $'BA. R^a :: 'AC. 'B,$ ^{a 32.}
therefore $'BA \times 'B^b = 'AC \times R.$ ^{b 16. c. 6.}
And because $'DA. R^a :: 'AC. 'D,$ ^{c 21. I.}
therefore $'DA \times 'D^b = 'AC \times R.$ ^{d 14. c. 6.}
therefore $'DA \times 'D^c = 'BA \times 'B$ and $'DA.$
 $'BA^d :: 'B. 'D. Q. E. D.$

Prop. XXXIX.

Prop. XXXIX. Fig. 22.

THe T^s of the Sides BC , DC , bc , dc ,
are reciprocally as σ^s vertical L^{tes}
 BCA , DCA , bca , dca .

Because $\text{BC} \cdot \text{AC}^2 :: \text{R} \cdot \sigma \text{BCA}$,
^a 33. therefore $\text{BC} \times \sigma \text{BCA}^b = \text{AC}$
^b 16. e. 6. $\times \text{R}$. and because $\text{DC} \cdot \text{AC}^2 ::$
^c ax. 1, $\text{R} \cdot \sigma \text{DCA}$, therefore $\text{DC} \times \sigma$
 $\text{DCA}^b = \text{AC} \times \text{R}$, therefore $\text{BC} \times \sigma \text{BCA}$
 $= \text{DC} \times \sigma \text{DCA}$ and $\text{BC} \cdot \text{DC}^b :: \sigma \text{DCA}$
 $\cdot \text{BCA}$. *Q. E. D.*

Prop. XL. Fig. 22.

THe Sines of the Sides BC , DC , bc ,
 dc are as those of their opposite
 L^{tes} D , B .

Because $\text{BC} \cdot \text{R}^2 :: \text{AC} \cdot \text{B}$,
^a 34. therefore $\text{BC} \times \text{B}^b = \text{R} \times \text{AC}$
^b 16. e. 6. and because $\text{DC} \cdot \text{R}^2 :: \text{AC} \cdot \text{D}$,
^c ax. 1. therefore $\text{DC} \times \text{D}^b = \text{R} \times \text{AC}$
^d 14. e. 6. therefore $\text{BC} \times \text{B}^c = \text{DC} \times \text{D}$
and $\text{BC} \cdot \text{D}^d :: \text{DC} \cdot \text{B}$, *Q. E. D.*

Prop. XLI

Prop. XLI. Fig. 23.

IN any Triangle ABC , as the rectangle under the Sines of the Sides CF or $MF \times AE$ is to the Square of the Radius $ON :: IL = IA - AL$ the Difference of the versed Sines of the Base AC and of the Difference of the Sides $AM = BM - BA$ to GN the versed Sine of the Arch PN , *i. e.* of the L^{ic} B .

On B as a Pole describe the great Circle PN , and $\parallel$ to it CM , their Plans PON , CFM will be right to the Plan BON , and PG , CH Perpendiculars to that Plan BON^c will fall on the common Sections ON , FM , suppose at G and H . The Isosceles

^a 20.
^c 38. e. II.
^d def. 6. e. II
^e 4. e. 6.
^f conv. II.
e. II.

Triangles PON , CFM are similar, for $L^{\text{ic}} PON^d = CFM$, therefore $FM. ON^e :: CM. PN^e :: MH. GN$. Draw CIL to A , then is $HI^f LOA$. The Triangles AEO , DFO , DIH , DLM are similar, for the Angles at E , F , I , L are right, and the Sides are either parallel or the

G same

50 Trigonometry. Part IV.

same Line continued, therefore $AE \cdot AO$
 $:: IL \cdot MH$, and before we had $FM \cdot ON$
 $:: HM \cdot GN$,

Therefore $AE \times FM \cdot AO \times ON :: IL \times$
 $HM, HM \times GN :: IL \cdot GN. Q. E. D.$

Prop. XLII. Fig. 24.

THe Difference of the versed Sines
of 2 Arches AC, AB viz. $HD \times \frac{1}{2}R =$

$$S_{\frac{1}{2}}AC + \frac{1}{2}AB \times S_{\frac{1}{2}}AC - \frac{1}{2}AB = AG \times BF.$$

$$\text{Diff. s. } v = AH - AD = HD.$$

$$s_{\frac{1}{2}} \text{ Sum Arches} = {}^sAI = AG. \text{ and}$$

$$s_{\frac{1}{2}} \text{ diff. Arches} = {}^sBI = BF.$$

The $\triangle^{les} AGO, BEC$ are similar, for

the Angles at E and G^a are

right, and $BC \parallel AG$, and BE

$\parallel AO$; and $AO \cdot AG^b :: BC$.

BE , therefore $\frac{1}{2}AO \cdot AG^c ::$

$\frac{1}{2}BC = BF. BE = HD$, there-

fore $\frac{1}{2}AO \times HD^d = AG \times BF. Q. E. D.$

Prop. XLIII.

A Rectangle under the versed Sine of
any Arch and $\frac{1}{2}R =$ to the Square
of

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of the Sine of $\frac{1}{2}$ that Arch. $\frac{1}{2}ob \times eb = ac_q$.

For the $\Delta^{1ca} ocb$, aeb are similar, having $L^c b$ common, and the Angles at c and e right, therefore $ob. cb^a :: ab. eb$, then $\frac{1}{2} ob. cb :: \frac{1}{2} ab = ac. eb$. therefore $\frac{1}{2} ob \times eb^b = ac_q$. Q. E. D.

Given.	Sought.	Proportions.	Fig. 25.
1 $L^c B. L^c D, BC. op.$	$C L^c vert.$	$\sigma BC. R^a :: \gamma B. BCA.$ $\sigma B. BCA_b :: \sigma D. sDCA$ $L^c BCD = BCA$ $\dagger DCA, \text{ or } BCA$ $- DCA, \text{ according as } B, D \text{ are}$ affected. by 27.	a 35. Inv. b 36.
2 B, C, BC $L^c L^c l.$	$D. op.$ L^c	$\sigma BC. R^c :: \gamma B. BCA.$ $sBCA. sDCA. d :: \sigma B. \sigma D$ If $BCA < BCD$ the $L^c D$ is $\times$ affected as B , else not.	c 35. Inv. d 36. x 27.

	Given.	Sought.	Proportions.
• 33. Inv. • 37.	3 BC. DC. B l. l. L^{le}	DB.	$R. \sigma B^{\text{e}} :: {}^{\text{t}}BC. {}^{\text{t}}BA.$ $\tau BC. \sigma BA^{\text{f}} :: \sigma DC. \sigma DA.$ $DB = BA^{\frac{1}{2}} AD$, ac- cording as the L falls within or without $\hat{y}$ Δ^{le} which is known by $L^{\text{le}} D$.
	4 BD. BC l. L^{le}	DC.	$R. \sigma B^{\text{e}} :: {}^{\text{t}}BC. {}^{\text{t}}BA.$ $\tau BA. \sigma BC^{\text{f}} :: \sigma DA. \sigma DC.$ $DC > \text{or} < 90^{\circ}$. ac- cordingly as $L^{\text{le}} D$ & ACD are affected by 24. and 25.
k 38.	5 B. D. BC $L^{\text{le}} L^{\text{le}} l.$	BD.	$R. \sigma B^{\text{e}} :: {}^{\text{t}}BC. {}^{\text{t}}BA.$ ${}^{\text{t}}D. {}^{\text{t}}B^{\text{k}} :: {}^{\text{s}}BA. {}^{\text{s}}DA.$ $BD = BA^{\frac{1}{2}} AD$, as $L^{\text{le}} B D$. are affect- ed. by 27.
	6 BC. BD. B l. l. L^{le}	D L^{le}	$R. \sigma B^{\text{e}} :: {}^{\text{t}}BC. {}^{\text{t}}BA.$ ${}^{\text{s}}DA. {}^{\text{s}}BA^{\text{k}} :: {}^{\text{t}}B. {}^{\text{t}}D.$ $L^{\text{le}} D$ is affected as B or not. as $BA > \text{or}$ $< BD$ by 27.

Given.

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Given.	ough.	Proportions.
7 B C. D C. B l. l. L ^{le}	C vert.	R. σ BC ^m :: τ B. γ B C A τ DC. τ BC ⁿ :: σ B C A. σ D C A
		L ^{le} C = B C A $\pm$ D C A as L falls within or with- out the Δ^e which is known by the Species of L ^{le} D.
8 B. C. B C. L ^{le} L ^{le} l.	D C.	R. σ BC ^m :: τ B. γ B C A σ D C A σ B C A ⁿ :: τ B C. τ D C.
		D C > or < 90°. as C D A. A C D, or A C, A D are affe- cted by 24. 25.
9 B C. D C. B l. l. L ^{le}	D.	σ D C. σ B C :: σ B C. σ D. Am- biguous.
10 B C. B. D. l. L ^{le} L ^{le}	D C.	σ D. σ B C :: σ B. σ D C. Am- biguous.

Given.

	Given.	Sought	Proportions.
q 41. r 42.	I 1 AB. BC. AC. <i>all the sides.</i>	B L^c Fig. 23	$s ABX^s BC. R^2 q :: IL. GN.$ $s \frac{1}{2} AC + AM X s \frac{1}{2} AC - AM$ $= IL X \frac{1}{2} R. \text{ And } NG X$ $\frac{1}{2} R^2 = s \frac{1}{2} NP^2 = s \frac{1}{2} L^c B.$
s 17. Fig. 17.	I 2 A. B. C. $L^c L^c L^c$	B C	The Sides of the Δ^{le} DEFY are the Complements of the opposite Angles in ABC: So the Case is the same as in preceeding.

Intro-

Introduction

TO THE Use of Both GLOBES.

HAVING already spoke of Circles on a Sphere in general, we shall now speak in particular of such Circles as are for ordinary marked on the artificial Globes; which Circles represent these imaginary ones on the natural Globe, with relation to which we calculate the Places and apparent diurnal and annual Motions of the Heavenly Bodies, and by Means of which we account for the Phenomina depending on these Motions. That this Earth is globular (abstracting from that very inconsiderable Difference of the Diameter and Axis of the Equator) appears in the Figure of its Shadow in a lunar Eclipse: And it is plain to any that can look about him
that

that the Heavens or starry Frame is a Sphere, whose Center is the Eye of the Beholder, or (to please the *Ptolomeans*, if there be any) the Center of this Earth: For once then we shall conceive the Heavens and Earth to be Two concentric Spheres, and then whatever Circles we imagine in the Heavens, we may conceive Circles in the same Position with relation to each other on this Earth; so the same Circles may be describ'd both on the Celestial and Terrestrial Globes.

The more remarkable great Circles on the Sphere are these :

- | | |
|----------------------|-------------------------------|
| 1. <i>Equator.</i> | 5. <i>Equinoctial Colure.</i> |
| 2. <i>Meridian.</i> | 6. <i>Solstitial Colure.</i> |
| 3. <i>Ecliptick.</i> | 7. <i>Vertical Circles.</i> |
| 4. <i>Horizon.</i> | 8. <i>Hour Circles.</i> |

I. The *Equator* is a great Circle passing from East to West, and having the same Poles and Axis with those of the World.

It is the Measure of the Sun's apparent diurnal Motion, for in the Space of a Day (*i. e.* 24 Hours) the Sun seems

to

Use of Both Globes. 57

to describe this Circle or a Circle parallel to it. Hence,

1. 15° of the Equator answer to an Hour of Time, and 1° to $4'$. And therefore,

2. To reduce Degrees of the Equator to Time: Divide by 15 and multiply the Remainder by 4, the Quot gives Hours and the Product Minutes.

The same way are Minutes of the Equator reduced to Minutes and 2^{ds} of Time.

II. *Hour-Circles* are great Circles passing through the Poles of the World, and every 15° of the Equator.

N. B. The Point directly above our Head is called the *Zenith*; and that directly below our Feet, the *Nadir*.

III. The *Meridian* is a great Circle passing thro' the Zenith and Nadir and the Poles of the World.

It is a moveable Circle, for it shifts its Place as the Zenith does, so every Point in the Globe has its own Meridian; and therefore the artificial Globe is hung in a Brazen Circle, which

H passes

passes thro' the Poles of the Sphere, that any Point of the Sphere being brought to that Circle, it may represent the Meridian of that Point.

For the same Reason Geographers have fixt upon one Meridian, from which they reckon the Distances of all other Meridians, and this they call the *First Meridian*.

Definitions for the Terrestrial Globe.

IV. *Latitude* is the Distance of a Place from the Equator, and is either North or South.

V. *Longitude* is the Distance of the Meridian of any Place from the first Meridian. It is always reckon'd Eastward.

VI. The *Distance* of Two Places is an Arch of a great Circle intercepted betwixt these Two Places.

The Brazen Meridian is divided into 4 Quadrants, the Two upper Quadrants are divided into 90° , and reckon'd from the Equator to the Poles; the other 2 Quadrants are reckon'd from the Poles to the Equator.

On

On the Two upper Quadrants are reckon'd the Latitudes of Places.

The Equator is divided into 360° , reckoned from the first Meridian Eastward compleatly round the Globe; on it are marked the Longitudes of Places.

VII. The *Ecliptick* is a great Circle cutting the Equator at Angles of $23^{\circ} 30'$; it is the Way of the Sun's apparent annual Motion.

VIII. The *Ecliptick* is divided into 12 equal Parts called *Signs*, each Sign contains 30° for $\frac{360^{\circ}}{12} = 30$.

The Sun describes this Circle 360° in the Space of 365 Days, and consequently he moves through almost 1° in one Day.

IX. The Time of the Sun's Revolution round the *Ecliptick* is called a *Year*.

X. The Year is divided, as the *Ecliptick* is, into 12 Parts almost equal, called *Months*.

The Signs with their correspondent Months and Characters stand thus:

♈	♉	♊	♋	♌
Aries,	Taurus,	Gemini,	Cancer,	Leo,
March,	April,	May,	June,	July,
♍	♎	♏	♐	
Virgo,	Libra,	Scorpio,	Sagittary,	
August,	September,	October,	November,	
♑	♒	♓		
Capricorn,	Aquary,	Pisces.		
December,	January,	February.		

XI. The *Horizon* is that Circle described by our Eye, when placed on a high Mountain, or when at Sea, we think we see the Heavens and Earth join, and this is called the *Sensible Horizon*.

XII. A great Circle parallel to this is called the *Rational Horizon*, its Poles are the Zenith and Nadir.

The rational Horizon is represented by the Wooden Horizon on the artificial Globe, on it are marked

1. A Circle divided into Signs and Degrees, as the Ecliptick is, beginning with the first Degree of ♈ at the East Point of the Horizon, and reckoning Northward.

2. A double Kalendar, viz. the *Julian* and *Gregorian*, each divided into Months

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Months and Days answering to the Divisions of the former Circle, beginning the *Julian* Kalendar with *March* 11th, at the East Point of the Horizon, and the *Gregorian* with *March* 22d, at the same Point, and reckoning both Northward.

3. A Circle divided, as the Sea Compass is, into 32 equal Parts called Rumbs, each Rumb contains $11^{\circ} 15'$ for $\frac{360^{\circ}}{32} = 11^{\circ} 15'$. Fig. 26.

XIII. The Elevation of the Pole is its Altitude above the Horizon; it is measured on the lower Quadrants of the Brazen Meridian.

The Elevation of the Pole is equal to the Latitude of the Place.

For making z the Place whose Horizon HR , QE the Equator, and P, p the Poles, QZ^a is the Latitude, PR the Elevation of the Pole, and it is plain $ZR^b = 90^{\circ}$ and $ZR - PZ = PQ$ Fig. 27,
a 4.
b2. Trig.P.4,
 $= PZ$. i. e. $PR = QZ$. Q. E. D.

When

When the Sun is on the Equator (i. e. at either Point where the Ecliptick cuts the Equator) the Days and Nights are equal all the World over.

For it is plain $QO = OE$, that is, $\frac{1}{2}$ the Equator is in Darkness, the other in Light. Hence the 1° of γ and α are called the *Equinoctial Points*.

Fig. 28. XIV. A *Right Sphere* is, when the Equator is at right Angles with the Horizon,

This Position of the Sphere they have that live at the Equator. The Poles of the World coincide with the North and South Points of their Horizon and their Days and Nights are equal all the Year over: For the Equator and all its Parallels are bisected by the Horizon.

Fig. 29. XV. A *Parallel Sphere* is when the Equator EQ coincides with the Horizon HR , and consequently all its Parallels are parallel to the Horizon.

This Position of the Sphere is proper to those that live at the Poles, and so the Poles of the World coincide with their

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their Zenith and Nadir. Seing the Sun can at once illuminate but Half the Globe, supposing him on the Equator at *E*, he then shines precisely to both Poles, and the Equator *E Q* and all its Parallels *r* *u*, *t w*, &c. have just one Half in Light, and the other Half in Darknes; that is, the Days and Nights are every where equal.

Supposing now the Sun at *x*, *viz.* 10° , North from the Equator, he shall then shine to *y*, 10° beyond *p*, & to *v*, 10° short of *p*; and describing a Circle upon *p* as a Pole parallel to the Equator, and passing through *y*; the whole Tract lying within that Circle shall have the Sun continually in View, and consequently they shall have continued Day. It is plain, that the Tract lying within the like Parallel passing through *v* shall have continued Darknes.

In like Manner the Sun being at *r*, the Point of his greatest Declination Northward from the Equator, *viz.* $23^{\circ} 30'$, he shall shine to *c*, $23^{\circ} 30'$ Southward from the Pole *p*, and to *A*, $23^{\circ} 30'$ short of *p*; and

and all that Tract lying within the Parallel AC , hath the Sun constantly in View, &c.

Thus from the Time of the Sun's being on the Equator at E , to the Time of his greatest Declination at r , the Sun has been continually in View of p , and p has been still in Darknes; that is, for the Space of 3 Months, answering to 3 Signs $\nu \propto \pi$, for the first Degree of $\propto$ is the Point of the Sun's greatest Declination Northward.

The Pole p has the Sun still in View during the following 3 Months, while he describes the 3 Sines $\propto \Omega \pi$, for all this while he is on the North-side of the Equator, and so shines beyond its North Pole. So the Inhabitants of the North Pole, if there are any, have had Half a Year's Day, and those of the South Pole Half a Year's Night, viz. from March 10th to September 13th.

The Sun, at $1^\circ \propto$, is again on the Equator, and the Days and Nights are again equal; thence he declines from the Equator Southward, and leaves the North

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North Pole in continued Darkness, while he gives $\frac{1}{2}$ Year's Light to the South Pole.

The Point of the Sun's greatest Declination Southward is the 1° of ϖ .

XVI. The Two Points of greatest Declination are called the Solstitial Points, and we having North Latitude, 1° $\odot$ is called our Summer Solstice, and 1° ϖ our Winter Solstice.

XVII. A great Circle passing thro' the Poles of the World and the Solstitial Points is called the *Solstitial Colure*.

XVIII. A great Circle drawn thro' the Poles of the World and Equinoctial Points is called the *Equinoctial Colure*.

XIX. A Circle drawn thro' 1° $\odot$ and parallel to the Equator, is called the *Tropick of Cancer*, for the Sun at that Point begins to return to the Equator.

XX. A Parallel to the Equator drawn thro' 1° ϖ is called the *Tropick of Capricorn*.

XXI. An *Oblique Sphere* is when the Equator $E Q$ and all its Parallels make
I oblique

Fig. 30. oblique Angles with the Horizon HR or hr .

This Position of the Sphere they have that live betwixt the Equator and the Poles. The Pole nearest to the Zenith is always elevated above the Horizon, and the other depreſt below it; The Elevation or Depreſſion of the Pole is always equal to the Latitude of the Place. Hence,

The Distance between the Poles of the World and thoſe of the Horizon is equal to the co-latitude of the Place; likewise the Elevation of the Equator above the Horizon on one Side, and its Depreſſion below it on the other is equal to the co-latitude.

1. Let the Sun be on the Equator at E , whatever the Horizon be; whether HR or hr , ſtill the Semidiurnal Arch $EO = 90^\circ = OQ$ the Semi-nocturnal Arch; that is, the Sun being on the Equator, the Days and Nights are equal all the World over. Make now the Horizon HR .

2. Sup-

2. Supposing the Sun at b , then is the Semi-diurnal Arch $bu > by$, but by and EO are similar Arches, then the Day, when the Sun is at b , is longer than when at E . Again, the Sun being at T the solstitial Point, the Semi-diurnal Arch $Tm > Tv$ i.e. $Tm > bu$, i.e. the Days increase as the Sun approaches nearer to the Zenith & *contra*.

It is plain Tm is the greatest Semi-diurnal Arch, and consequently the Days are at the longest when the Sun is on the solstitial Point next to the Zenith.

3. The greater the Latitude is, the greater the longest Day is.

Make the Horizon hr , where pr the Elevation of the Pole = Lat. is greater than PR the Elevation of the Pole above the Horizon HR , it is plain the Semi-diurnal Arch $Tm > TM$.

4. If the Latitude $EZ = 66^\circ 30'$ the Complement of the Sun's greatest Declination, their longest Day shall be 24 Hours, for the Elevation of the Equator $EH = \text{co-lat.} = 23^\circ 30'$ the Sun's greatest Declination,

Fig. 31.

tion, therefore the Horizon HR coincides with the Ecliptick, and the Semi-diurnal Arch TR coincides with the Tropick of Cancer, and so touches ; but does not cut the Horizon.

The Poles of the Meridian are the East and West Points of the Horizon. Hence, the Horizon is a movable Circle ; for the Meridian being movable, its Poles are so too, and with them the Horizon moves.

For this Reason the Globe is so set in the Wooden Horizon, that any Point of the Globe may be brought to the Vertex or Zenith, and so it represents the Horizon of that Place.

The Poles of the Equinoctial Colure are the Two Points where the solstitial Colure cuts the Equator & *vicissim*.

The Poles of the Ecliptick are Two Points in the solstitial Colure as far distant from the Poles of Equator as the Ecliptick declines from the Equator.

XXII. Two Circles parallel to the Equator describ'd by one Revolution of the Poles of the Ecliptick round the Globe,

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Globe, are called *Polar Circles*. That describ'd by the North Pole of the Ecliptick is called the *Arctic*, the other the *Antarctic* Polar Circle, as A C, &c.

The Two Tropicks with the Two Polars divide the Globe into Five Zones.

XXIII. That lying betwixt the Two Tropicks is called the *Torrid Zone*, as being expos'd to the direct Stroke of the Sun's Rays, and consequently to the most violent Heats.

XXIV. The two Zones lying within the Polars are called the *Frigid Zones*, the Stroke of the Sun's Rays being there very oblique, and consequently faint and weak.

XXV. The Two Zones lying betwixt the Tropicks and Polars are called *Temperate Zones*, as partaking equally of the Heats of the Torrid and Colds of the Frigid Zones.

The Inhabitants of the several Zones take their Names from the various Projections of their Shadows, when the Sun is on their Meridians.

XXVI. The

XXVI. The Inhabitants of the Torrid Zone are called *Amphiscii*, as having their Shadows projected sometimes North, sometimes South, according as the Sun is on the South or North Side of their Zenith.

To these the Sun is vertical twice a Year, and then they are *Ascii*, having no Meridian Shadow.

To these that live under the Tropick, the Sun is vertical but once a Year.

XXVII. These in the Frigid Zones are called *Periscii*; for they having the Sun still in View, their Shadows describe a Circle once in 24 Hours.

XXVIII. The Inhabitants of the Temperate Zones are called *Heteroscii*, as having their Shadows projected only one Way.

Supposing Parallels of Latitude to be drawn through every Degree and Minute of the Meridian, which we shall call simply *Parallels*, and calling the Two opposite Semi-circles of the same Meridian, as they are cut off by the Axis of the World, *opposite Meridians*, we may

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may then divide the Inhabitants of the Globe into *Antæci*, *Periæci* and *Antipodes*.

XXIX. *Antæci* have the same Meridian, but opposite Parallels; and consequently they have the same Days and Nights, but opposite Seasons and opposite Poles elevated.

XXX. *Periæci* have the same Parallel, but opposite Meridians: They have the same Seasons and the same Pole elevated, but opposite Days and Nights.

XXXI. *Antipodes* have opposite Parallels and opposite Meridians: They have opposite Seasons, opposite Days and Nights, and opposite Poles elevated.

XXXII. *Climates* are little Zones terminated by Two Parallels of Latitude, so taken as the longest Day at the greater Latitude is Half an Hour more than the longest Day at the lesser.

The longest Day at the Equator is 12 Hours, and at either Polar it is 24 Hours, the Number of *Climates* then betwixt the Equator and either Polar is 24, and their Number betwixt the Two Poles is 48. The

The longest Day at either Pole is 6 Months, and allowing the Days to increase by Months from the Polars to the Poles, there shall then be 6 Climates from each Polar to its nearest Pole, in both 12, which added to the former 48 give in all 60 Climates.

If the longest Day be supposed to increase by $\frac{1}{4}$ Hour, these Zonulæ are called Parallels.

Because the right and left Parts of Heaven are frequently mentioned, we shall here observe that

1. Philosophers and Geographers called the *East* the *right*, and the *West* the *left* Parts: The former, because the Heavenly Bodies apparently moved from East to West: The latter, because in observing the Poles Altitude the East was on their right Hand.

2. Astronomers reckoned West the right and East the left Parts: For they, looking Southward in observing the Motions of the Stars, had the West on their right, and East on their left Hand.

3. Augurs

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Use of Both Globes. 73

3. Augurs, looking toward the rising Sun, reckoned South the right, and North the left Parts.

4. Poets, looking towards the setting Sun, reckoned North the right and South the left Parts.

Hence, *Ovid. Met. Lib. 2.*

*Neu te dexterioꝛ tortum declinet in anguem;
Neve sinistroꝛ pressam rota ducat ad aram.*

Here too, having formerly mentioned both Kalendars, we shall shew how to find the *Golden Number*, *Epact* and the *Dominical Letter* for the *Julian Kalendar*.

XXXIII. The *Golden Number* is the Space of 19 Years the Time of one Revolution of the Moon's *Nodes* (*i. e.* the Two Points where her Orbit cuts the *Ecliptick*) round the *Ecliptick*.

XXXIV. The *Epact* is the Space of 11 Days the Excess of the Solar above the Lunar Year; the former containing 365 and the latter 354 Days.

To find the *Golden Number* and *Epaēt* for any Year.

$$\begin{array}{r}
 1713 \\
 +1 \\
 \hline
 19)1714(90 \\
 \quad 4 \text{ G.N.} \\
 \quad 11 \\
 \hline
 30)44(1 \\
 \quad 30 \\
 \hline
 \quad 14 \text{ Ep.}
 \end{array}$$

$$\begin{array}{r}
 \text{Epaēt } 14 \\
 \text{July } 5 \\
 \text{Day } 14 \\
 \hline
 33 \\
 -30 \\
 \hline
 \text{Moon's Age } 3
 \end{array}$$

So 3 is the Moon's Age July 14th 1713.

To the Year given add Unity, divide that Sum by 19, the Remainder is the *Golden Number*.

Multiply the *Golden Number* by 11, divide the Product by 30, the Remainder is the *Epaēt*.

So 4 is the *Golden Number* and 14 the *Epaēt* for the Year 1713.

To the *Epaēt* add the Day of the Month, and Number of Months from *March* inclusive; that Sum (if less than 30) is the Moon's Age: If more, the Excess is the Moon's Age.

To

Use of Both Globes. 75

To find the Change of the Moon for any Month.

To the *Epaēt* add the Number of Months from *March inclusive*; subtract that Sum from 30, or if more than 30 from 60, the Remainder is the Time of Change.

$$\begin{array}{r} \text{Epaēt } 14 \\ \text{August } 6 \\ \hline 20 \\ \hline 30 - 20 = 10 \end{array}$$

So the Moon changes *August 10th*, 1713.

N. B. The *Epaēt* for any Year does not reckon till *March*.

To find the Time of the Moon's coming to the Meridian.

Multiply the Moon's Age by 4, divide the Product by 5; the Quot gives Hours; the Remainder multiplied by 12 gives Minutes.

For 12° being the Moon's mean Motion from the Sun in one Day; that multiplied by 4 (*viz.* 4 x 12°) gives the same Arch in Minutes of Time; and making *x* the Moon's Age or Num-

$$\begin{array}{r} 11 \\ 4 \\ \hline 5) 44(8^h 48' \\ 40 \\ \hline 4 \\ 12 \\ \hline 48 \end{array}$$

K 2

ber

ber of Days from the Conjunction, at which Time the Sun and Moon were at once on the Meridian, $4 \times 12 \times 3$ shall give the Moon's Distance from the Sun in Minutes of Time, which divided by 60 ($= 5 \times 12$) gives $\frac{4 \times 12 \times 3}{5 \times 12} = \frac{4}{5}$ the same Distance in Hours; and what remains shall be Fifth Parts of an Hour, *i.e.* 12'.

So on the 11th Day of the Moon's Age she is on the Meridian at 8 H. 48'.

Hence, 1. To find the Time of high Water at any Port, the Time of high Water at the Full or Change being given.

The Sum of the Time of the Moon's southing and that of high Water at the Change, gives the Answer.

So at London high Water at the Change is at 3, therefore at the 11 Day of the Moon it is $3 \text{ H} + 8 \text{ H. } 48' = 11 \text{ H. } 48'$.

2. To find the Hour of the Night by the Moon shining on a Sun-Dial.

To the Time of the Moon's southing add the Distance of the Hour on the Dial

Use of Both Globes. 77

Dial from 12, if the Shadow falls amongst the Afternoon Hours; if not, subtract.

XXXV. In the Kalender is a continued Revolution of the first 7 Letters of the Alphabet, representing the 7 Days of the Week; by these we may find the Feria or Day of the Week answering to any Day of the Year, provided we know which of these Letters represents *Sunday*, or is the *Dominical* or *Sunday* Letter for that Year.

To find the *Dominical* Let. for any Year.

Divide the Year given by 4, (the Remainder gives the Year after Leap-Year, and if 0 remain it is Leap-Year)

to the Dividend

add the Quot and the Number 5:

Divide that Sum by 7, subtract the Remainder from

8, what remains gives the Number

of Letters from A.

So D is the *Do-*

$$\begin{array}{r} 4 \overline{) 17132} \\ \underline{428} \end{array} \left. \vphantom{\begin{array}{r} 4 \overline{) 17132} \\ \underline{428} \end{array}} \right\} 1 \text{ remains.}$$

$$\begin{array}{r} 7 \overline{) 21462} \\ \underline{306} \end{array} \left. \vphantom{\begin{array}{r} 7 \overline{) 21462} \\ \underline{306} \end{array}} \right\} 4 \text{ remains.}$$

$$\begin{array}{r} 8 - 4 = 4 \end{array}$$

1. 2. 3. 4.

A. B. C. D.

minical

78 Introduction to the

minical Letter for the Year 1713, being the first after Leap-Year.

N. B. Because of the Intercalation of a Day in Leap-Year at the 6th. *Cal. Mart.* (*i. e.* February 24th;) Leap-Year has Two Letters, the first reckoning from 1st *January* to February 24th, the other for all the rest of the Year.

To find the *Dominical* Letters for Leap-Year.

$$\begin{array}{r} 4)1720\} \\ 430\} \end{array} \text{o remains.}$$

$$\begin{array}{r} 5 \\ 7)2155\} \\ 307\} \end{array} \text{6 remains.}$$

$$\begin{array}{r} 8-6=2 \end{array}$$

I. 2.

A. B.

ing Leap-Year.

The Letters answering to the first Day of every Month in the Kalender are the same with the first Letters of the Words in this Distich.

Find the Letter as if it were a common Year, and prefix to it the Letter immediately after it in the natural Order of the Alphabet. So C B are the *Dominical* Letters for the Year 1720, be-

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Gratia,
July,

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Use of Both Globes.

79

*Astra, Dabit, Dominus, Gratiſque, Beabit, Egenos,
January, Feb^r. March, April, May, June,
Gratia, Chriſticola, Feret, Aurea, Dona, Fideli.
July, Auguſt, Septemb^r. Octob^r. Nov^r. Decemb^r.*

Given the *Domi-
nical* Letter, to find
what Day of the
Week any Day of a-
ny Month is.

Find the Day of
the Week answering
to the Letter of the
firſt Day of the
Month ; that Day
ſhall be the 1, 8, 15,
22, 29 Days of that
Month : The other
Days are eaſily found.

The ſame may be
done by this Kalen-
der. Thus,

Find the Month
on the Top, the Let-
ter immediately be-
low it answers to
the firſt Day of that

Month : Find the *Dominical* Letter on
the

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
January October	A	B	C	D	E	F	G
May	B	C	D	E	F	G	A
August	C	D	E	F	G	A	B
Feb. March November	D	E	F	G	A	B	C
June	E	F	G	A	B	C	D
September December	F	G	A	B	C	D	E
April July	G	A	B	C	D	E	F
1	2	3	4	5	6	7	
8	9	10	11	12	13	14	
15	16	17	18	19	20	21	
22	23	24	25	26	27	28	
29	30	31					

the left Hand, the Day of the Week above the first Days Letter gives the first Day of the Month. Find the first Day's Letter on the left Hand, the Letter opposite to it and directly above the given Day of the Month is the Letter for that Day.

XXXVI. *Vertical Circles* are great Circles passing thro' the Zenith and Nadir, and any Point of the Globe taken at Pleasure.

They are represented by the Quadrant of Altitude, which is a Brazen Quadrant that is screwed to the Zenith, and is moveable compleatly round the Globe.

XXXVII. Circles passing thro' the Poles of any great Circle are called *Secondaries* to that Circle.

So Verticals are Secondaries to the Horizon, and Meridians are Secondaries to the Equator.

Probl. I.

TO rectify the Globe for the Latitude of any Place.

Elevate the Pole to the Latitude of the Place, and Screw the Quadrant of Altitude to the Zenith.

Problems on the Terrestrial Globe.

Probl. II.

TO find the Longitude and Latitude of a given Place.

Bring the Place to the Brazen Meridian, the Degree of the Meridian above the Place is its Latitude, the Degree of the Equator cut off by the Meridian is its Longitude.

Probl. III.

Given the Longitude and Latitude of a Place, to find that Place on the Globe.

L

Bring

Bring the Longitude to the Brazen Meridian, the Place is below the Latitude.

Probl. IV.

TO find the Distance of Two Places.

Rectifie the Globe for the Latitude of one of these Places, bring the Quadrant of Altitude to the other, the Arch intercepted betwixt the Two Places gives the Distance in Degrees, which multiplied by 60, gives the Distance in Geometrick Miles.

Definitions for the Celestial Globes.

XXXVIII. *Longitude* is the Distance of the Sun or Star from the first Degree of γ , reckon'd on the Ecliptick.

XXXIX. *Latitude* is the Star's Distance from the Ecliptick reckoned on the Secondaries to the Ecliptick, it is either North or South.

Nota, The Sun has no Latitude.

XL. *Declina-*

XL. *Declination* is the Distance of the Sun or Star from the Equator, reckoned on the Secondaries to the Equator. It is either North or South.

XLI. *Right Ascension* is the Degree of the Equator, cut off by the Horizon when the Sun or Star is brought to the Horizon in a right Sphere : Or, because in all Positions of the Sphere the Equator is still perpendicular to the Meridian,

Right Ascension is the Degree of the Equator, cut off by the Brazen Meridian, when the Sun or Star is brought to that Meridian in any Position of the Sphere.

XLII. *Oblique Ascension* is the Degree of the Equator cut off by the Horizon, when the Sun or Star is brought to the Horizon in an oblique Sphere.

XLIII. *Ascensional Difference* is the Difference betwixt the right and oblique Ascension.

N. B. If the Latitude of the Place and Declination of the Sun or Star be towards the same Part, *viz.* both North

or both South, the right Ascension is the greater & *contra*.

XLIV. The Sun or any Star is said to rise when it comes to the East side of the Horizon: And to set when it comes to the West side of the Horizon.

The Poets have a three-fold rising of the Stars, *viz.* *Cosmic*, *Achronic* and *Heliac*.

XLV. A Star is said to rise *Cosmically* when it rises with the Sun. So *Virg. Geo. L. 1.*

Candidus auratis aperit cum cornibus annum Taurus, &c. i. e. When the Sun enters *Taurus*, and they both rise at once.

A Star sets *Cosmically* when it sets at Sun-rising or rises at Sun-setting. So *Virg. Geo. L. 1.*

*Antè tibi Eoæ Atlantides Abscondantur
Debita quàm sulcis committas semina, quamq;
Invita properes anni spem credere terra.*

Harvest-Time is meant when the Sun being in *Scorpio*, the *Pleiades* set *cosmically*.

XLVI. A Star is said to rise *Achronically*, when it rises at Sun-setting.

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Use of Both Globes. 85

*Et Careo vobis scythicas detrusus in oras,
Quatuor autumnos Pleias Orta facit. Ovid.*

The Sun being in Scorpio, the Pleiades rise achronically and set cosmically.

A Star is said to set achronically when it sets with the Sun: So *Ovid. fast. L. 2.*
*Quem modò cælatum stellis delphina videbas
Is fugiet visus nocte sequente tuos.*

February the 3^d, When the Sun being in Aquary, the Delphine sets achronically.

Hence, the Sign, in which the Sun is, rises cosmically, and sets achronically, and the opposite Sign, at Night rises achronically, and in the Morning sets cosmically.

XLVII. The *Heliac* rising of a Star is when the Star appears in View after having been for some time lost in the Sun's greater Light. So *Ovid. fast. L. 2.*
*Jam levis obliqua subsedit Aquarius urna
Proximus athereos excipe Piscis equos*

About the latter End of February.
*Ante tibi Eoæ Atlantides abscondantur,
Gnosque ardentis decedat stella corona,
Debita quam sulcis committas semina, &c.*
Virg. G. 1.
The

The Sun being in *Scorpio*, the *Pleiades* set cosmically, and the *Corona Septentrionalis* arise heliacally.

XLVIII. The heliac setting of a Star is when the Sun's greater Light obscures that Star.

Candidus auratis aperit cum cornibus annum Taurus, & adverso cedens canis occidit astro.

XLIX. *Amplitude* is the Distance of the rising or setting Sun or Star from the East or West Point of the Horizon. It is either *Ortive*, when the Star rises, or *Occidual*, when it sets.

L. *Altitude* is the Elevation of the Sun or Star above the Horizon, reckoned on the Secondaries to the Horizon.

LI. *Azimuth* is the Distance betwixt the North Point of the Horizon, and that Point where a vertical, passing through the Body of the Sun or Star, cuts the Horizon.

Hence, *Verticals* are called *Azimuths* or *Azimuth Circles*.

Probl. V.

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Probl. V.

TO find the Sun's Place on the Ecliptick at any Time.

In the Kalendar on the Wooden Horizon find the Day of the Month, and opposite to it, on the Circle divided as the Ecliptick, you shall find the Sign and Degree the Sun is in. Find that Sign and Degree on the Ecliptick on the Globe, so you have the Sun's Place on the Globe. So *April 3d* the Sun's Place is $\vee 20^{\circ} 30'$.

Prop. VI.

TO find the Sun's Declination and right Ascension.

Bring the Sun's Place to the Brazen Meridian, the Degree of the Meridian above the Sun's Place gives his Declination: The Degree of the Equator cut off by the Meridian, gives his right Ascension.

N. B. The Co-latitude of any Place is equal to the Sum or Difference of the Declina-

Declination and Meridian Altitude, according as the Latitude and Declination are toward contrary or the same Parts.

Probl. VII.

TO find the Sun's Ortive or occidual Amplitude, oblique Ascension and Ascensional Difference.

Rectifie the Globe to the Latitude of the Place, and bring the Sun's Place to the East side of the Horizon, the Distance betwixt the Sun's Place and the East Point of the Horizon gives his Ortive or occidual Amplitude, the Degree of the Equator cut off by the Horizon is his oblique Ascension; so the Ascensional Difference is easily found.

Probl. VIII.

TO find the Hour and Minute of the Sun's rising and setting, and the Length of the Day and Night.

Reduce

Use of Both Globes. 89

Reduce the Ascensional Difference to Time. So $6^h \pm$ that Time (according as the Latitude and Declination are towards the same or contrary Parts) gives the Time of the Sun's rising: And $6^h \pm$ that Time gives his setting; double the Time of his rising gives the Length of the Night; and double the Time of his setting gives the Length of the Day. Or thus,

Bring the Sun's Place to the Brazen Meridian, fasten the Horary Circle on that Meridian, with the Index pointing 12 of the Day, turn the Globe 'till the Sun's is on the East side of the Horizon, the Index shall point the Hour of Sun-rising, &c.

Probl. IX.

TO find the Beginning of the Morning and Ending of the Evening Twi-Light.

Bring the Point of the Ecliptick, opposite to the Sun's Place, to the West side of the Horizon, move the Globe

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and

and Quadrant of Altitude at once, till that Point be elevated 18° above the Horizon: the Difference betwixt the Point of the Equator on the East Point of the Horizon, and the oblique Ascension reduced to Time, and subtracted from the Time of Sun-rising, gives the Beginning of the Morning Twi-Light: In like Manner may the End of the Evening Twi-Light be found.

- *Probl. X.*

TO find the Sun's Altitude and Azimuth at any Time of the Day: Or his Altitude given, to find his Azimuth and the Hour of the Day.

1. Reduce the Hour from 12 given, to Degrees of the Equator; bring the Sun's Place to the Brazen Meridian, and turn round the Globe, 'til that Arch of the Equator pass the Meridian; so the Quadrant of Altitude laid on the Sun's Place, shall cut off his Azimuth on the Horizon,

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Horizon, and shew his Altitude above his Place.

2. Bring the Sun's Place to the Brazen Meridian; Move the Globe and Quadrant of Altitude at once, 'til the Sun's Place be on the given Altitude, the Difference betwixt the right Ascension and the Degree of the Equator cut off by the Meridian, gives the Hour from 12; and the Quadrant cuts off the Azimuth on the Horizon.

The Two last Problems may be done by Help of the Horary Circle.

The Declination, right and oblique Ascensions, Amplitude, Altitude and Azimuth of such fixt Stars as rise and set are found after the same Manner.

Probl. XI.

TO find the Longitude and Latitude of the fixt Stars.

Elevate the Pole to the Altitude of $66^{\circ} 30'$, bring the Pole of the Ecliptick to the Brazen Meridian, screw the Quadrant of Altitude to the Zenith, and

M 2

bring

bring it to the Star proposed, the Degree of the Quadrant above the Star gives its Latitude, and the Arch of the Ecliptick cut off by the Quadrant gives its Longitude.

N. B. All Stars, whose Declination is towards the same Parts with the Latitude of the Place and less than its Co-latitude, rise and set, otherways they never set.

Probl. XII.

TO find the Continuation of any fixt Star above the Horizon.

$6^h \pm$ the Star's Assensional Difference, according as its Declination is towards the same or contrary Parts with the Latitude of the Place, gives $\frac{1}{2}$ its Continuation above the Horizon.

Probl. XIII.

Probl. XIII.

GIVEN the right Ascensions of the Sun and any fixt Star, to find the Time of that Star's Culminating, *i. e.* of its coming to the Meridian.

The fixt Star's right Ascension ($+360^\circ$ if need be) — the Sun's right Ascension gives the Distance from 12 of the Day to the Time of Culminating.

Probl. XIV.

GIVEN the Time of Culminating of any Star and $\frac{1}{2}$ its Continuation above the Horizon, to find the Time of that Star's rising and setting.

The Time of Culminating — $\frac{1}{2}$ its Continuation gives its rising, and $+\frac{1}{2}$ its Continuation gives its setting.

Probl. XV.

Probl. XV.

TO find the Degree of the Ecliptick which rises or sets with a given Star, and to find the Time of that Star's rising or setting cosmically and achronically.

Bring the Star to the East side of the Horizon, seek the then rising Point of the Ecliptick on the Wooden Horizon, and in the Kalendar adjoined to it you have the Time of the Star's rising cosmically: The then setting or descending Point of the Ecliptick gives the Time of the Star's rising achronically, &c.

Probl. XVI.

TO find the Heliac rising and setting of any Star.

Bring the Quadrant of Altitude to the West side of the Meridian, the given Star to the East side of the Horizon and

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and the 12th Degree of Altitude on the Quadrant to the Ecliptick, the Point of the Ecliptick below that Degree is elevated 12° above, and its opposite Point deprest 12° below the Horizon; that opposite Point on the Wooden Horizon shews in the Kalendar the Time of the heliac rising of any Star of the first Magnitude.

The Converse of this Operation gives the Time of the heliac setting.

If the Star be of the second Magnitude, allow 13° of Depression; if of the third, 14° , &c.

Plain Dialing

By the GLOBES.

DIALS take their Names from the Circles of the Sphere to which their Plans are parallel; or with whose Plans they coincide.

I. The

I. The *Hour-Lines* are the common Sections of the Hour-Circles of the Sphere with those Plans.

II. The *Stile* or *Gnomon* is a Line parallel to the common Section of the Plans of all the Hour-Circles of the Sphere, *i. e.* parallel to the Axis of the World. Hence,

The Elevation of the Stile above the Dial-Plan must be always equal to the Elevation of the Axis of the World above the Plan of the Circle to which the Dial-Plan is parallel; and the Stile must always be directed towards the elevated Pole.

III. A *Horizontal* Dial is whose Plan is parallel to the Horizon.

IV. A *Vertical* Dial is whose Plan coincides with that of some vertical Circle.

V. A *Direct Erect* vertical Dial is whose Plan is perpendicular to the Horizon, and whose Face looks directly South or North.

VI. An *Erect declining* vertical Dial is whose Plan is perpendicular to the

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Horizon, but its Face declines from South or North Eastward or Westward.

VII. A *Direct reclining* Dial is whose Plan looks directly South or North, but falls back from the Zenith.

VIII. A *Direct reclining* vertical Dial whose Reclination is towards the same Parts with the Latitude of the Place, and equal to the Co-Latitude, is called a *Polar Dial*, its Plan passing through the Poles of the World.

IX. A *Direct inclining* vertical Dial is whose Plan looks directly South or North; but makes an acute Angle with the Horizon.

X. If the Latitude of the Place and the Inclination of the Plan be towards contrary Parts, and the Inclination equal to the Co-Latitude, that Dial is called an *Equinoctial Dial*.

N. B. Every Dial-Plate having Two Faces, it is plain the upper Face of the Equinoctial Dial shall have its Reclination equal to the Latitude; and the under Face of the Polar Dial an Inclination equal to the Latitude.

N

XI. From

XI. From what has been said, it may be easily known what is meant by *reclining declining*, and *inclining declining* vertical Dials.

XII. An *Erect vertical* Dial whose Face looks directly East or West, is called a *Meridian Dial*.

XIII. The *Substilar* is the Line in the Dial-Plan, upon which the Stile is set: It is the common Section of the Dial-Plan and the Plan of a great Circle passing thro' the Poles of the World and those of the Plan.

Probl. I.

Fig. 32.

TO find the Inclination of a Plan.

Let AB be a Plan inclin'd to the Horizon HK , apply to the Plan AB a Quadrant DCF , so as the Plummet CE may strain the Surface of the Quadrant, I say the Arch DE is the Measure of the L^{ic} of Inclination ABH . Draw BGL HR , because $CE \parallel BG$, the $L^{\text{ic}} ECF = CBG$, but $DCF = GBH$ both being
right

Use of Both Globes. 99

right L^{ic} , the $L^{ic} DCF - ECF = GBH - CBG$, that is, $DCE = ABH$. *Q.E.D.*

Probl. II. Fig. 33.

TO find the Reclination of a Plan.

Let AB be the reclining Plan. Draw $BGLHR$, representing the prime vertical; so ABG is the Angle of Reclination. Raise $KL \perp AB$, apply a Quadrant CDF to KL ; so as the Plum-Line CE may strain the Face of the Quadrant, I say DE is the Measure of the Angle of Reclination ABG .

In the right angled Triangle NKB the $L^{ic} BNK \downarrow NBK =$ a right Angle $= DCF$: But, because $CE \parallel BG$, the $L^{ic} ECF = BNK$, therefore $DCE = NBK$. *Q.E.D.*

Probl. III. Fig. 34.

TO find the Declination of any Plan.

N 2

Take

Take a Piece of Board, whose upper Surface is a right angled Parallelogram as DB , on it describe a Circle Cxz ; on the Center C erect a pin perpendicularly. Draw $FG \parallel BE$, $M \parallel LB$, $LN \perp FG$, and place the plan DB horizontally with the side BE applied to the Dial-Plate: Observe when the Shadow of the Top of the Pin is on the Periphery of the Circle as at z in the Forenoon, and at x in the Afternoon the same Day; bisect the Arch xz with the Diameter KL , 'tis plain KL is the Meridian, and $KLN = KM$ is the Angle of Declination.

Probl. IV.

TO find the Distances of the Hour-Lines on a Horizontal Plan.

It is plain the Brazen Meridian represents the Line of 12.

Rectifie the Globe to the Latitude of the Place, bring the Equinoctial Colure to the Brazen Meridian, turn the Globe West.

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Westward, 'till 15° of the Equator pass the Meridian; the Arch of the Horizon intercepted betwixt the Meridian and Colure, is the Measure of the Angle form'd by the Line of 12, and the Line of 1 at the Center of the Horizon: Again turn the Globe Westward, 'till 30° of the Equator pass the Meridian, so you have the Angle of the Lines of 12 and 2 as before.

The same Hour Distances serve for the Forenoon.

The Substilar is the same with the Line of 12, and the Elevation of the Stile is equal to the Latitude of the Place.

Probl. V.

TO find the Hour-Distances on a direct erect vertical Plan.

Rectify the Globe to the Latitude of the Place, bring the Equinoctial Colure to the Brazen Meridian, and the Quadrant of Altitude to the East Point of the

the Horizon, turn the Globe Eastward, 'till 15° of the Equator pass the Meridian, the Arch of the Quadrant intercepted betwixt the Meridian and Colure is the Measure of the Angle of the Line of 12, and Line of 11 at the Center of the vertical Circle, &c.

The same Hour-Circles serve for Afternoon. The Substilar is the same with the Line of 12; and the Elevation of the Stile equal to the Co-Latitude.

Direct reclining vertical Dials are the same with direct erect vertical Dials, in a Latitude equal to the Latitude of the Place $\pm$ the Reclination according as the Reclination is towards the same or contrary Parts with the Latitude.

The same holds in direct inclining vertical Dials.

Probl. VI.

TO find the Hour-Distances on an erect declining vertical

Plan

Plan : Declining from South Westward, suppose 25° .

Rectify the Globe to the Latitude of the Place, bring the Quadrant of Altitude to 25° of the Horizon (reckoning from East toward South) and bring the Equinoctial Colure to the Brazen Meridian; turn the Globe Eastward, 'til 15° of the Equator pass the Meridian; the Arch of the Quadrant intercepted betwixt the Colure and Meridian is the Measure of the Angle made by the Line of 12 and Line of 1 at the Center of the Declining Plan, &c.

2. Bring the Quadrant to 25° of the Horizon from West Northward, and the Equinoctial Colure to the Brazen Meridian; turn the Globe Westward, till 15° of the Equator pass the Meridian; so you have, as before, the Hour-Distance from 12 to 1, &c.

3 Keep the Quadrant in the same Position, bring the Equinoctial Colure to 25° from South Westward, the Arch of the Quadrant cut off by the

Colure

Colure gives the Angle made by the Substilar and Line of 12; and the Arch of the Colure intercepted betwixt the Quadrant and Pole of the World gives the Elevation of the Stile.

We shall refer declining reclining, and declining inclining vertical Dials, 'til we come to the Projection of the Sphere, as being more easily done that way.

Probl. VII.

Fig. 35.

TO describe a Polar Dial.

Draw the Horizontal Line *mk*, upon which raise the Perpendicular *D 12* to any Length; upon *12* as a Center with *12 D* as a Radius, describe a semicircle *N D L*, which divide into 12 equal Parts; through the Center *12* and each Division, draw Lines *12 E*, *12 F*, &c. So you have the Hour Points *E, F, G, K*, &c. on the Horizontal Line *mk*; Lines drawn thro' these Points, parallel to *12 D* are the Hour-Lines.

A Pin, erected at D perpendicular to the Plan and equal to $12 D$, is the Gnomon, pointing the Hour with the Shadow of its Top.

Or a Plate erected perpendicularly on $12 D$ as a Substilar, having its Altitude equal to $12 D$, and its upper side $\parallel 12 D$ is the Stile.

Probl. VIII. Fig. 36.

TO describe a Meridian Dial.

Draw the Horizontal Line MB , and make the Angle MBP equal to the Co-Latitude of the Place, so BP is the common Section of the Equator and Meridian Plan, raise $DC \perp BP$, and in it take any Point C for a Center, and on it, with CD as a Radius, describe a Semi-Circle NDL , which divide into 12 equal Parts, so you may have the Hour-Points $E, F, \&c.$ in the Line BP ; the Hour-Lines are Lines drawn thro' these Points parallel to CD .

O

The

The Stile is a Pin, equal to CD , erected perpendicular to the Plan at D , as in the Polar Dial.

The Problems of the Sphere may be solved by Spherique Trigonometry; by it too we may calculate the Hour-Distances on the several Dial-Plans; which Calculation we shall refer to the Projection of the Sphere on the Plan of the Horizon.

The SOLUTION of most Problems of the Sphere by Spherique Trigonometry.

Lemma I. Probl. I.

TO find the Sun's Place in the Ecliptick.

Keep

Use of Both Globes. 107

Keep in Mind the Days on which the Sun enters the several Signs, which are these,

[♈] Mar. 10. [♉] Ap. 9. [♊] May 10. [♋] Jun. 11.
[♌] Jul. 12. [♍] Aug. 12. [♎] S. 12. [♏] O. 12. [♐] N. 11
[♑] D. 11. [♒] Jan. 9. [♓] F. 8.

If, at the Day given, the Sun be in the Sign proper to the given Month; subtract the Day the Sun enter'd that Sign from the Day given; multiply the Remainder into 59' 8".

So you have the Degree of that Month's Sign the Sun is in. So *August* 25 the Sun is in [♍] 12° 48' 44" for 25 — 12 = 13 and 13 x 59' 8" = 12° 48' 44.

°	'	"
59	8	
	13	
2	57	24
1	31	20
8	20	
12	48	44

O 2

If

If the Sun be in a Sign different from the proper Sign of the Month, from

$$\begin{array}{r}
 31 \\
 - 12 \\
 \hline
 19 \\
 + 6 \\
 \hline
 25 \\
 \hline
 0^{\circ} 59' 8'' \\
 \hline
 25 \\
 4 \ 55 \ 40 \\
 3 \ 02 \ 40 \\
 16 \ 40 \\
 \hline
 \Omega \ 24 \ 38 \ 20
 \end{array}$$

the Number of Days contain'd in the preceeding Month subtract the Day of the Sun's entring into that Month's Sign, to the Remainder add the Day of the Month given; that Sum multiplied into $59' 8''$ gives the Degree the Sun is in.

So *August 6.* the Sun is in *Leo* $24^{\circ} 36' 20''$ for $31 - 12 + 6 \times 59' 8'' = 24^{\circ} 38' 20''$.

Lemma II. Probl. II.

Given the Sun's Place, to find the nearest Equinoctial Point.

Divide the Ecliptick into 4 Quadrants, thus:

- | | | | |
|-------|----|----|----|
| 1. Q. | ♈. | ♉. | ♊. |
| 2. Q. | ♋. | ♌. | ♍. |
| 3. Q. | ♎. | ♏. | ♐. |
| 4. Q. | ♑. | ♒. | ♓. |

While

Use of Both Globes. 109

While the Sun is in the first and last Quadrants, γ is the nearest Equinoctial Point, and when in the middle Two $\approx$.

In the first Quadrant the Sun's Longitude is the Distance from the nearest Equinoctial Point.

In 2d Q. $180^\circ - \odot$'s Longitude is the Distance.

In 3d Q. $\odot$'s Longitude $- 180^\circ$ is the Distance.

In 4th Q. $360^\circ - \odot$'s Longitude is the Distance.

Probl. III. Fig. 37.

TO find the Sun's Declination and right Ascension.

Let $PEPQ$ be the solstitial Colure, EQ the Equator, $\textcircled{S}$ $\textcircled{W}$ the Ecliptick, $\odot$ the Sun's Place, γ $\odot$ or $\approx$ $\odot$ his Distance from the next Equinoctial Point; PDp a Meridian passing through $\odot$ the Sun's Place, 'tis plain $\textcircled{S} \gamma \textcircled{S}$. $\textcircled{S} \gamma \odot^a ::$

$\textcircled{S} E. \odot D$, that is R. Dist. $\odot ab$

Equinoct. :: greatest Decl. to the present Declination.

^a 28. Trig.
P. 4.

2. $\textcircled{S} E \textcircled{S}$.

^b 29. 2. $\text{E}\text{S}.\text{D}\text{O}^b :: \text{VE}.\text{vD}$ that is 'greatest Declination. 'Declination ::

R. Dist. *ab Equinoct.* on the Equator.

In the 1st Quadrant vD is the right Ascension.

In 2d Q. $180^\circ - \text{vD}$ is the right Ascension.

In 3d Q. $180^\circ + \text{vD}$ is the right Ascension.

In 4th Q. $360^\circ - \text{vD}$ is the right Ascension.

Probl. IV. Fig. 38.

TO find the Sun's ortive or occidual Amplitude, and his Ascensional Difference.

Let ZHN be the Meridian of the Place H R its Horizon, EQ the Equator, c v L the Ecliptick, $\odot$ the Sun's Place; draw the Meridian $\text{P}\odot\text{p}$, that cuts off the right Ascension at d , and F the Point of the Equator that comes to the Horizon with the rising Sun cuts of the oblique Ascension, so Fd is the Ascensional Difference, and $\text{F}\odot$ the Amplitude;

it

it is p
i.e. 'c
censio
:: R. '1

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$\frac{x \times R}{Z \odot x}$

$\frac{1}{2} \text{co-de}$

$\frac{x \times X}{\text{co-alt.}}$

Use of Both Globes.

III

it is plain $\text{'R Q. 'O } d^a :: \text{'R. 'F } d,$
i.e. $\text{'co-Lat. 'Decl. :: \text{'R. 'Af-}$
 $\text{fensional Diff and 'R Q. 'O } d^b$
 $:: \text{'R. 'F O.}$

^a 29.

^b 28.

Probl. V. Fig. 39.

Given the Sun's Altitude and Declination, and the Latitude of the Place, to find his Azimuth.

In the Triangle $P \odot Z$ we have given $P \odot$ the Co-Declination, $Z \odot$ the Co-Altitude, and PZ the Co-Latitude, sought the Angle $PZ \odot$ the Azimuth.

$$\frac{{}^s \frac{1}{2} P \odot + Z \odot \sim PZ \times {}^s \frac{1}{2} P \odot - Z \odot \sim PZ}{\frac{1}{2} R} = x$$

$$\frac{x \times R^2}{{}^s Z \odot \times {}^s PZ} = z \text{ And } z \times \frac{1}{2} R = \frac{{}^s \frac{1}{2} PZ \odot^2}{2} \text{ that is,}$$

$$\frac{{}^s \frac{1}{2} \text{co-decl.} + \text{co-alt.} \sim \text{co-lat.} \times {}^s \frac{1}{2} \text{decl.} - \text{co-alt.} \sim \text{co-lat.}}{\frac{1}{2} R} = x$$

$$\frac{x \times R^2}{\text{co-alt.} \times {}^s \text{co-lat.}} = Z. \text{ And } Z \times \frac{1}{2} R = \frac{{}^s \frac{1}{2} \text{Azim.}^2}{2}$$

Probl. VI.

Probl. VI. Fig. 39.

Given the Sun's Altitude, Azimuth and Declination, to find the Hour of the Day in any given Latitude.

In the Triangle PZO , $\angle POZ$ $\angle PZO :: \angle ZOZ$ the Measure of the Hour from 12.

^a 40. Trig.
P. IV.

Probl. VII. Fig. 40.

TO find the Beginning of the Morning and Ending of the Evening Twi-Light.

The Depression GO and Declination dO being known, NO and pO are easily found, and pN is equal to the Co-Latitude of the Place, sought the Angle OPN , the Measure of the Hour from Mid-Night.

$S \frac{1}{2} NO \perp$

Use of Both Globes. 113

$$\frac{{}^s\frac{1}{2}\text{NO} + \text{PO} \sim \text{PN} \times {}^s\frac{1}{2}\text{NO} - \text{PO} \sim \text{PN}}{= x}$$

$$\frac{1}{2}\text{R}$$

$$\frac{x \times \text{R}^2}{{}^s\text{PO} \times {}^s\text{PN}} = z \text{ And } z \times \frac{1}{2}\text{R} = \frac{{}^s\text{R}}{2} \sim \text{PN}^2 \text{ that is,}$$

$$\frac{{}^s\frac{1}{2}\text{co-depr.} + 90 \div \text{decl.} \sim \text{co-lat.} \times {}^s\frac{1}{2}\text{co-dep.} - 90 \div \text{dec.} \sim \text{col.}}{= x}$$

$$\frac{1}{2}\text{R}$$

$$= x, \text{ \&c.}$$

Probl. VIII. Lemma. Fig. 41.

TO find the Declination of any Star by Observation.

Let HR be the Horizon, EQ the Equator, P the Pole of the World, Z the Zenith, s , σ , Σ or f the Star, observe the Star's Meridian Altitude HS , $\text{H}\sigma$, Rf , $\text{R}\Sigma$, 'tis plain $\text{HS} - \text{HE} = \text{ES}$ the Declination North, $\text{HE} - \text{H}\sigma = \text{E}\sigma$, $\text{R}\Sigma - \text{RP} = \text{P}\Sigma$ the Co-Declination, and $\text{Rs} + \text{RQ} = \text{Qs}$ the Declination.

Probl. IX. Lemma.

Given the Sun's right Ascension, to find the right Ascension
 P
cension

cension of any fixt Star by Help of a Pendulum Watch.

When the Sun is on the Meridian, set your Watch at 12, observe the Hour and Minute when the Star comes to the Meridian, that Time reduced to Degrees of the Equator, is the Difference of their right Ascensions.

Probl. X. Fig. 42.

Given the Declination and right Ascension of any Star, to find its Longitude and Latitude.

Make $E Q$ the Equator, $c L$ the Ecliptick, P and Z their Poles and s the Star, its Declination $A S$ being given, $P S$ is known and $90^\circ - \angle A$ the right Ascension $= E A = L^e Z P S$.

In the $\Delta^e Z P S$, $R. \sigma Z P S^e ::$

^a 33. Trig.

^b 38. Trig.

^c 40. Trig.

$'P S. 'x, 'P Z + x. 'x^b :: 'Z P S. 'Z S P.$

$'Z S P. 'Z P^c :: 'Z P S. 'Z S$ the Co-

Latitude. Also $'Z S. 'Z P S^c ::$

$'P S. 'P Z S$ the Co-Longitude.

Probl. XI.

Probl. XI. Fig. 43.

TO find the Ascensional Difference of any fixt Star, and consequently its oblique Ascension and Continuation above the Horizon.

It is plain oA is the Ascensional Difference, and s being the Star on the Horizon HR , and PSA being the Secondary to the Equator EQ , $\angle RQ$. $\angle SA :: R$, $\angle OA$, *i. e.* $\cot$ Lat. $\angle Decl. :: R$. Ascensional Difference.

That Ascensional Difference subtracted from the right Ascension of the Star, if it declines toward the elevated Pole, or added to it, if otherways, gives the oblique Ascension.

The same Ascensional Difference added to or subtracted from 90° , according as the Declination is towards the elevated or deprest Pole gives the Semi-diurnal Arch of that Star.

In the same Triangle the ortive Amplitude os is easily found.

Probl. XII. Fig. 44.

Given the oblique Ascension of any Star, to find its cosmick and achronick rising.

Let s be the rising Star. In the $\triangle^{ic}$ $\nu O A$, we have given νO the oblique Ascension, the $L^{ic} O \nu A = 23^\circ 30'$ and $E O A$ the L^{ic} of the Equator with the Horizon. And $R. \nu O :: \nu O A. \cot. x$
 $\sigma O \nu A. + x. \sigma x :: \nu O. \nu A$, so A the Point of the Ecliptick that rises with the Star is known, when the Sun then comes to that Point the Star rises cosmically, and when he comes to the opposite Point he rises achronically. In the same Triangle the $L^{ic} \nu A O$ is easily found.

Probl. XIII. Fig. 44.

TO find the heliac rising and setting of any Star.

Let D be the Sun's Place while the Star s rises heliacally. In the Triangle $A D F$ right angled at F , we have given $F D$ the Depression and the $L^{ic} F A D$ that

the Ecliptick makes with the Horizon,
and $\angle FAD$. $\angle FD :: R$. $\angle AD$, so $\angle A + AD$
 $= \angle D$ gives the Sun's Place when the
Star rises or sets heliacally.

Spheric Geometry.

Containing

The Stereographic and
Orthographic Projections of the Sphere.

Definitions.

- I. **S**tereographick Projection represents the several Circles of the Sphere, on a Plan passing thro' its Center, in such Sort as they would appear to the Eye were it placed on the Surface of the Sphere, and at the Pole of that Circle on whose Plan the other Circles are projected.
- II. That

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II. That Circle, on whose Plan the other Circles are projected, is called the *Primitive Circle*.

N. B. It is plain that all the Rays that pass from the Eye to all the Points of the Periphery of any Circle, whether parallel or oblique to the Primitive, form a *Cone*, whose Base is that Circle, its Vertex the Eye, and its Axis a right Line joining the Eye and Center of the Circle; and consequently if the Circle be parallel to the Primitive the Cone is right; if not, it is oblique.

III. The *Line of Measures* of any Circle is that Line in the Primitive Circle, in which the Center of that Circle, when projected, is found.

Lemma I. Fig. 45.

IF a Cone $ABOC$ be cut by a Plan passing thro' its Vertex A and the Diameter of its Base BC , the Section shall be a Triangle.

ad. 18. e. 11. For the Diameter BC is a right Line, and so are AB , AC , for the Cone may be describ'd by either.
Ergo, &c. *Cor.*

Cor
joinin
Base,

IF a
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shall
M
is a F
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Diam
ting t
AF.
& FC.
Again
and B
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where

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Δ^{le} th

Spheric Geometry. 119

Cor. Hence the Axis, being a Line joining the Vertex and Center of the Base, is in the Plan of the Triangle.

Lemma II. Fig. 45.

IF a Cone $ABOC$ be cut by a Plan parallel to its Base the Section $ekgh$ shall be a Circle.

Make F the Center of the Base, then is AF the Axis, and ABC a

Triangle; make $eg \parallel BC$ the ^a *Lem. prec.* Diameter of the Section, cutting the Axis at i , draw $ik \parallel OF$, then is ^b *4. e. 6.*

$AF.BF^b::Ai.ie \& ex aq uo FC.FB::ig.ie$

$\& FC.AF^b::ig.Ai$ But $FC=FB$ then $ig=ie$.

Again $AE.OF^b::Ai.ik \& ex aq uo BF.OF::ei.ik$.

and $BF.AF^b::ei.Ai$

But $BF=OF$, therefore $ei=ik=ig$.
wherefore $ekgh$ is a Circle.

Lemma III. Fig. 46.

IF a scalenous Cone $ABKC$ be cut by a Plan $EHD \perp ABC$, the Plan of the Δ^c thro' the Axis, and in a sub-contrary Posi-

Position to the Base, *i. e.* so as the Δ^u ADE be similar to ABC *viz.* the L^c AED $= L^c$ ACB, &c. the Section EHD is a Circle.

Draw the Plan FHGLABC the Plan of the Δ^c and $\parallel$ BKC, so^a is FHG a Circle: The common Section of the Plans EHD, FHG, HIL^bFG and ED. Now the Δ^{les} ABC, ^cADE, ^dAFG are similar; therefore the Δ^c FIE is similar to DIG, and DI. IG^c::FI. IE. and DI X IE^f=FI X IG^g=IG^g, therefore EHD^g is a Circle. Q. E. D.

^a L. 2.

^b 19. e. 11.

^c Hyp.

^d sch. 4. e. 6.

^e 4. e. 6.

^f 16. e. 6.

^g Cor. 12.

e. 6.

Prop. I. Fig. 47, 48.

Every Circle on the Sphere, whether parallel or oblique to the Primitive, is a Circle in the Projection.

Make BC the Circle to be projected on the Plan of the Primitive EG, at whose Pole A suppose the Eye to be placed; then shall B be projected into *b*, and C into *c*; and because AD=AC the

$$L^c ACD =$$

$L^{\text{le}} A C D = A B C$, therefore the $\Delta^{\text{le}} A B C$ is sub-contrarily cut by $D C$ or $b c$, therefore $b c$ is a Circle. *Q. E. D.*

Prop. II.

Fig. 49.

THE Distance of the Center of any great Circle from the Center of the Primitive is equal to the Tangent of its Elevation above the Plan of the Primitive.

Suppose the Eye at A , projecting the Circle $B C$ on the Plan of the Primitive $E G$, the Point B appears in the Line of Measures at b and $F b = T^{\frac{1}{2}} B F H$ the Point c appears at c , and $F c = T^{\frac{1}{2}} H F C$ and $B A c$ is a right Angle. Bisect $b c$ the projected Diameter at i , that is the Center of the projected Circle $A b H$ and $F i = B F E$ the Elevation of $B C$ above the Primitive.

For, 'cause $A F^{\text{c}} \perp b c$, the $\Delta^{\text{les}} b A F$, $F A c$, $b A c^{\text{d}}$ are similar, and $\frac{1}{2} B F H^{\text{c}} = b A F$, $\therefore A c F^{\text{c}} = \frac{1}{2} A i F$, therefore $B F H = A i F$,
Q there-

^a d. 10. P. 1.

Trig.

^b 31. e. 3.

^c d. 6. P. 4.

Trig.

^d 8. e. 6.

^e 20. e. 3.

therefore their Complements are equal,
viz. $BFE = FAi$ whose Tangent is
 Fi . *Q. E. D.*

Cor. 1. The Radius of the projected
 Circle is equal to the Secant of its Ele-
 vation above the Plan of the Primitive
 $bi = Ai$.

2. bF the Distance of the projected
 Circle from the Center of the Primitive
 on the Line of Measures $= bAF = \frac{1}{2}$
 BFH the Co-Elevation.

3. Set off $Hm = BE$, draw the Dia-
 meter lm , that shall be the Axis of BC
 and l, m , its Poles, join Am ; it is plain
 the Pole m shall appear in the Line of
 Measures at p and Fp its Distance from
 the Center of the Primitive $= FAp$
 $= \frac{1}{2}$ Elevation.

4. Hence, The Pole of any great
 Circle falls always betwixt the Center
 of that Circle and that of the Primitive.

Praxis in projecting oblique great
 Circles reckoning the Distances from the
 Center of the Primitive, then the Di-
 stance of the Center $=$ 'Elevation, the
 Distance of the Pole $= \frac{1}{2}$ Elevation, the
 Distance

Distance of the projected Circle on the Line of Measures = $\frac{1}{2}$ Co-Tangent and the Radius = Secant of the Elevation.

Prop. III. Fig. 50.

EVERY great Circle $\perp$ to the Primitive is projected into a right Line, and is measur'd on the Line of Semi-Tangents.

Suppose the Eye at A, projecting the Circle EHG upon the Plan of the Primitive EG, the Points H, o, p, k, m, appear to the Eye at F, a, b, d, e, therefore the whole Arch EHG shall be projected into the Line EG. But $Fa^2 = FAa = \frac{1}{2}HFo = Ho$. Q. E. D.

Prop. IV. Fig. 51.

THE Distance of the Center of a lesser Circle, whose Pole is in the Plan of the Projection, from the Center of the Primitive, is equal to the Secant of the Distance of that lesser Circle from its Pole, and its Radius is equal to the Tangent of that Distance.

Q 2

Let

Let the lesser Circle BC be to be projected by the Eye at A , upon the Plan EG , in which Plan the Pole G of the lesser Circle is found. It is plain bc shall be the projected Diameter, which bisected gives i for the Center, $ib = iB$ for the Radius.

The $\Delta^{les} ABH, CFH$, having the L^{les} at aB and bF right and the $L^{le}H$ common, are similar, then we have $L^{le}FBb^c = FAB = FCB^c = iBc$; But $bBi \perp iBc$ $\therefore$ a right $L^{le} = bBi + FBb = FBi$, therefore Bi = the Radius bi , is the Tangent and Fi the Secant of BG . *Q. E. D.*

Prop. V. Fig. 52, 54.

TO describe a lesser Circle, whose Poles are not in the Plan of the Projection.

Let the Circle BC be to be projected, by the Eye at A , on the Plan EG ; bc shall be its projected Diameter, which bisected gives d for the Center. It is plain Fb, Fc are the Semi-Tangents of the

the Arches BH , CH , *i.e.* of the Distances of the Extremities of its Diameter from the Pole of the Primitive.

Prop. VI. Fig. 55.

IF Two Circles CBF , ABI , cut each other at B ; the $L^{ic} FBI$ made by the Circles $= L^{ic} ABC$ made by their Radii at the Point of Intersection B .

Draw the Tangents BD , BE . Because the infinitely small Parts of the Circles at B , coincide with BD , BE , therefore the curved Angle $FBI = DBE =$ the right $L^{ic} DBC - EBC =$ the right $L^{ic} EBA - EBC = CBA$. *Q. E. D.*

Prop. VII. Fig. 53.

ALL great Circles passing thro' the Point b have their Centers in the Line $ILLEI$ the Line of Measures to the Circle $AbHL$ Circle EG and passing thro' I its Center. I say the Center of the Circle QbR is in the Perpendicular IL as at K . For all great Circles
on

^e1 Trig. P. 4. on the Sphere cutting each other at Points diametrically opposite, their Representatives in the Projection must cut at the same projected Points, suppose b, c , now the Δ^{les} $\kappa ib, \kappa ic$ have the L^{es} at i right, $ib^2 = ic^2$ and $i\kappa$ common, therefore $\kappa b, \kappa c$ being equal are radii and κ the Center of the Circle QbR . Q. E. D.

Schol. The Distance $i\kappa$ of the Center κ from i the Center of the Perpendicular AbH is = the Tangent of the $L^{e} HbR$ answering to the Radius ib , for $i\kappa$ = the Tangent of the $L^{e} ib\kappa^d = L^{e} HbR$. Q. E. D.

Prop. VIII. Fig. 56.

THE Angles made by Circles on the Sphere are equal to the Angles made by their Representatives on the Plan of the Projection.

Suppose the Eye at A projecting the $L^{e} RBS$ on the Plan EF ; draw BD, BC Tangents to the Two Circles at B . The

The
and
Circ
Sect
at A
CF.
 $L^{e} D$
A Q
 $= A$
DB.
CD
and
 $= L$

IF a
H o,
these
PN, a
pose a
K Q
tercep
cle z
so the
ho to

The Plan DBC , made by the Tangents, and EF are both $\perp$ to the Plan of the Circle BQA , therefore their common Section DC is $\perp$ to that Plan. The Eye at A projects DB into DF , and CB into CF . I say the $L^{ic} DFC = L^{ic} DBC$. Draw $BQ \parallel EF$ join AQ . The $L^{ic} DBA^c = AQB^c = ABQ^c = BFD$, therefore $DB^g = DF$. In $\Delta^{ics} CDB$, CDF the L^{ics} at D are right; $DB = DF$ and CD is common, therefore $L^{ic} DFC = L^{ic} DBC$. $Q. E. D.$

^c 32. e. 3.
^d 5. e. 1.
^e 29. e. 1.
^g 6. e. 1.
^k 4. e. 1.

Prop. IX.

Fig. 57.

IF a great Circle $zPNp$ pass thro' the Poles of Two great Circles $\mathcal{A}Q$, HO , and if p, n , the remotest Poles of these Circles be join'd by a right Line PN , about which PN as an Axis suppose a Plan $PDKN$ to move, the Arches $KQDO$, of the Circles $\mathcal{A}Q$, HO , intercepted betwixt $PDKN$ and the Circle $zPNp$ shall be always equal, as also the Arches of any Two Parallels aq, ho to these Circles.

For

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For $PO = NQ$, $PD = NK$ and l^k
 $OPD = QNK$ being the Inclination of
the Two Plans, therefore $DO = KQ$
the same way $do = kq$. *Q. E. D.*

Prop. X. Fig. 58.

TO make the Scale of natural
Sines, Tangents, Secants,
Semi-Tangents and Chords.

Divide any Circle $A D B E$ into Four
Quadrants, and each Quadrant into 90°
tho' here we have only every 10° mark'd.

1. It is plain that Lines drawn from
each Degree of the Quadrant $A D L A C$
the Radius, shall mark the Sines of the
several Arches on that Radius
^{a 34. c. 1.} $A C$, thus nx cuts off $cx^2 =$
 $nf = 40^\circ$. So CA is the Line of Sines.

2. Draw EH touching the Circle at
 E , Lines drawn thro' the Center C and
every Degree of the Quadrant $A E$ shall
mark the Tangents of the several Arches
on the Line EH , and shall themselves
be Secants of these Arches, which Se-
cants, being transposed to CA produced,
give

gives CK for the Line of Secants, and EH is the Line of Tangents.

3. Thro' D and each Degree of the Quadrant EB draw right Lines; these shall cut off the Tangents of $\frac{1}{2}$ these Arches on the Line CB . So CB is the Line of Semi-Tangents.

4. Draw DB ; on D as a common Center with the Chords of the several Arches of the Quadrant DB as Radii describe Arches; these shall mark the Chords of the several Arches of the Quadrant DB on the Line DB ; so DB is the Line of Chords.

Prop. XI. Probl.

TO find the Poles of any Circle.

1. The Poles and Center of the Primitive coincide.

2. The Extremities of a Diameter L to a right Circle are its Poles.

3. $\frac{1}{2}$ r the Elevation of any oblique Circle, set off from the Center of the Primitive on the Line of Measures, gives the Pole of that oblique Circle.

R

Prop. XII.

Prop. XII. Probl. Fig. 59.

TO lay down on the Projection any Angle required.

1. If the Center of the Primitive be the angular Point the Case is the same as in plain Angles.

2. If the angular Point be in the Periphery of the Primitive as at *A*, thro' *A* and the Center *E* draw *AC*; on *A* and *C* as Centers with the Secant of the L^e sought, strike 2 Arches cutting one another at *x*, that is the Center of the Circle, and *HAC* the L^e sought.

3. Let *G* be the angular Point, and the L^e to be made by oblique Circles, that L^e , as *HGN*, shall be made by sch. pr. 7.

4. At a Point *G* in a right Circle *DEGF*, to make, with that Circle, an L^e of any Number of Degrees as 68° .

Draw *HEKJDF*, thro' *K* and *G* draw *KM*, set off $Ei = {}^T FM$ and $ILJDF = \cot. 68^\circ$ answering to the Radius *GI* upon *L* as a Center with the Radius

L G

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LG describe a Circle; it is done. For $L^e HGD = 90^\circ$ and $NGH = 22^\circ$ then $DGN = 68^\circ$. Q. E. F.

Prop. XIII. Probl. Fig. 60.

TO draw a great Circle passing thro' a given Point P, and making any L^e as $HQP = 50^\circ$, with the Primitive.

Upon the Center E with the Tangent of the given L^e strike an Arch; and upon the given Point P with the Secant of the same Angle strike another Arch; their Point of Intersection z^a is the Center and zP the Radius of the Circle sought. Q. E. F.

Prop. XIV. Probl. Fig. 61.

TO draw a great Circle passing thro' any Two Points F, G within the Primitive.

R 2

Thro'

Thro' either of the Points as F draw a Diameter AFEC; cross it at right L^{th} with the Diameter BD; join FB and draw BILFB and cutting AC produc'd at I, that Point I is a 3d Point, through which the Circle must pass.

We have here only to prove that the Line HK joining the Points of Intersection with the Primitive, is a Diameter, produce HEM 'til it meet the

^a 35. e. 3.
^b cor. 8. e. 6.

Circle at L, then EHXEL
^a = EFX EI ^b = EB_q = EHX
 EM, therefore EM = EL.
 Q. E. A.

Prop. XV. Probl. Fig. 62.

TO draw a great Circle L to a given great Circle.

Draw it thro' its Poles.

1. If the oblique Circle AKC must be LBD a right Circle and pass thro' a given Point K; thro' K and the Poles of BD, A, C, describe a Circle.

2. If the Perpendicular AKC must make a given L^{th} with the Primitive, as of

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26°, on the right Circle BD from the Center E , set off $Ex^a = \text{tang.}$ of the given L^c , or from the Pole A to x set off $Ax^b = \text{Se-}$ ^{a 2.} ^{b 1. Cor. 2.} cant of that Angle, so x is the Center.

3. If to the great Circle AKC a $\perp$ must be drawn so as to pass thro' a Point m given in the Circle AKC . Thro' P , the Pole of AKC , and m^c describe a Circle, and it is ^{c 14.} done.

4. If the Perpendicular FPG must make a given L^c , as of 62° , with the Primitive, 'tis done by 13th.

Prop. XVI. Probl.

TO measure any Arch of a great Circle.

1. The Primitive is measur'd on the Line of *Chords*.

2. Right Circles are measur'd on the Line of *Semi-Tangents*.

3. A Ruler laid thro' the Pole of any oblique Circle, and the Divisions of the Primitive, cuts off the like Divisions on the oblique Circle by 9th. Prop. XVII.

Prop. XVII. Probl. Fig. 63.

TO measure any Spherique Angle.

1. If the angular Point be in the Periphery of the Primitive, the Distance of the Center of the oblique Circle from that of the Primitive is the Tangent of the L^e .

2. If the Angle is within the Primitive; the Arch of the Primitive intercepted betwixt Two Lines drawn from the Angular Point thro' the Poles of the Circles forming the L^e is the measure of that Angle, so v and y being the Poles of the Circles ALC , OLQ , the Arch de is the Measure of the L^e QLC .

Prop. XVIII. Probl. Fig. 64.

TO measure any Arch of a lesser Circle.

On the Center of the Primitive with a Radius = the $\frac{1}{2}$ Tangent of the Distance

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Spheric Geometry. 135

stance of the lesser Circle, from its furthest Pole, describe a Circle thro' the Divisions of that Circle and the nearest Pole of the Lesser Circle, lay a Ruler that cuts off the like Divisions of the lesser Circle by 9th.

To Project the Sphere on the Plan of the solstitial Colure. Fig. 65.

With the Chord of 60° describe the Primitive, draw the Diameter pp , representing ^a the Equinoctial Colure and Axis of the World. Draw the Equator ^a 3. ^b 4. ^c 5. ^e 4. $EQIPp$, from E set off $Ew = 23^\circ 30'$ join $w\mathfrak{S}$ for the Ecliptick: On pp produced as a Line of Measures with a Radius ^b $= 66^\circ 30'$ describe the Tropicks $T\mathfrak{S}$, $t\mathfrak{w}$ on the same Line of Measures with a Radius ^a $= 23^\circ 30'$ describe the Polars Aa , Bb , set off $EH = 34^\circ = \text{Co-Lat. } Ea'$, join HR ^a for the Horizon of Ea' .

Find

Find the Sun's Place in the Ecliptick, as $\odot$, for any Time: Draw the Meridian $p o p$ cutting the Equator at x ; so $\odot x$ is the Declination and $v x$ the right Ascension, draw $D \odot N^b \parallel E Q$. So is $D S$ the Semi-diurnal, and $N S$ the Semi-nocturnal Arch; and $v s$ the ortive Amplitude; for $D \odot N$ cuts the Horizon at s .

Suppose the Sun's Altitude at any Time of the Day, his Place being $\odot$, to be 39° set off $H K = 39^\circ$ Draw $K^s L^b \parallel H R$ cutting $D \odot N$ at σ ; the Arch $D \sigma$ reduced to Time gives the Hour from 12, and drawing $Z \sigma N$ thro' Z, N , the Poles of the Horizon and σ , cutting $H R$ at m , $R m$ is the Sun's Azimuth.

To project the several Circles of the Sphere on the Plan of the Equinoctial Colure. Fig. 66.

THE Primitive, Equator, Tropicks and Polars are drawn as before $p p$ represents the solstitial Colure and Axis

Axis of the World on pp
 produced, set off $66^{\circ} 30'$, so
 you have the Center of the Ecliptick
 E & Q ; on the same pp , set off 56° so
 you have the Center of QHE the Ho-
 rizon of Ed : suppose the Sun's Place,
October 20, to be $\pi 8^{\circ}$ that is 38° from
 the nearest Equinoctial Point, set off
 $Q\gamma = 38^{\circ}$ and lay a Ruler thro' γ and
 π , the Pole of the Ecliptick, that cuts
 the Ecliptick at o the Sun's Place: Draw
 pop cutting the Equator at x , so Ex
 is the right Ascension, and ox the De-
 clination; thro' o and π , the Pole of
 pop , lay a Ruler that cuts off $Qm =$
 Declination; the Point where a Tan-
 gent drawn thro' m , cuts pp produced
 is the Center of the Parallel of Decli-
 nation mon , which cuts the Horizon
 at s, σ , so $s\sigma$ is the full diurnal Arch,
 and σE the ortive or occidual Ampli-
 tude.

To project the Sphere on the Plan of
the Horizon of Edinb^r Lat. 56° .

Fig. 67.

Draw the North and South Line
NS NS LU UE the prime vertical; the
Equator being elevated above the Pri-
mitive 34° , the North Semi-Circle of
the Ecliptick $57^{\circ} 30'$ and the South
 $10^{\circ} 30'$ are easily ^adrawn, and
^bare here represented by uqE ,
 $u\textcircled{S}E$, $u\textcircled{W}E$; the ^bPoint $\textcircled{S}$ is
one Extremity of the Diameter of the
Tropick of $\textcircled{S}$, from z on zN produced,
set off $\frac{1}{2}^{\circ} 100^{\circ} 30'$ ($= zP + P\textcircled{S} = 34^{\circ} +$
 $66^{\circ} 30'$) that gives the other Extremi-
ty of its Diameter. So $r\textcircled{S}\textcircled{S}$ is easily
drawn: The same way is the Tropick
of Capricorn drawn.

Let the Sun's Place be $8^{\circ} 8'$ find $\odot$ his
Place in the Ecliptick. Draw
 $p\odot x$ cutting the Equator at
 x , so $\odot x$ is the Declination,
and $v x$ the right Ascension. Draw ^bthe
Parallel of Declination $d\odot E$, so is dy
the

the Semi-diurnal Arch, ys the Meridian Altitude, and DU the ortive or occidual Amplitude. Take $\frac{1}{2}$ Co-Altitude for a Radius, and on z as a Center, strike an Arch cutting DOE at g , so is gy the Hour from 12, and cs the Azimuth from South.

By Means of this Projection plain Dialing may be perform'd, for the Hour-Lines being nothing else but the common Sections of the Hour-Circles with the Dial-Plans; having in this Projection describ'd the Hour-Circles and the Circle of the Sphere to which the Plan is Parallel, their common Sections give the Hour-Distances.

I. To make a Horizontal Dial.

Fig. 68.

THe common Sections of the Hour-Circles with the Horizon cut off the Hour-Distances $N1, N2, N3, \&c.$ from Ns the Meridian or Line of 12.

S 2

By

By Calcul. In the Spherique $\triangle^{\text{ic}} \text{NPI}$ right angled at N; $L^{\text{ic}} \text{NPI} = 15^\circ$ the Hour-Distance on the Equator, $\text{NP} = \text{Lat.}$ and R. $^{\circ}\text{NP} :: ^{\circ}\text{NPI. } ^{\circ}\text{NI. i.e. R. } ^{\circ}\text{Lat.} :: ^{\circ}\text{Hour-Distance on the Equator. } ^{\circ}\text{h. d. on Horizon.}$

II. To make a direct erect vertical Dial.

THe common Sections of the Hour-Circles with 6 & 6 the prime vertical, cut off $za, zb, zc, \&c.$ the Hour-Distances from N S the Line of 12.

Calcul. In the $\triangle^{\text{ic}} \text{PZA}$ right angled at z the $L^{\text{ic}} \text{ZPA} = 15^\circ$ and $\text{PZ} = \text{co-lat.}$ and R. $^{\circ}\text{PZ} :: ^{\circ}\text{ZPa. } ^{\circ}\text{za, } \&c.$

III. To make an erect declining vertical Dial, declining suppose from South Westward 25° .

SEt off $6d = 25^\circ$, join $Dz d$ that is the declining Circle; draw $2z2$ $\perp$ Dd , so $2, 2$, are the Poles of Dd join $2p2$ cutting Dd at 1 ; $2p2$ is the sub-stilar,

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stilar, pi the Elevation of the Stile, and iz the Distance of the substilar from the Meridian, the Arches zp , zq , zr , &c. give the Forenoon Hour-Distances, &c.

Calcul. In the Triangle $pi z$ right angled at i . $pz = \text{Co-Lat.}$ and $L^c pz i = \text{Co-Decl.}$ and $R. p z^2 :: p z i$.

$^a pi$. elev. Stile.

$^a 28.P.4.T.$

2. $\sigma pi. R^b :: \sigma pz. \sigma iz p.$

$^b 31.P.4.T.$

$^c 29.P.4.T.$

Dist. Substil. from Mer.

3. $^a pi. R^c :: iz. ipz. ipz + zp = ipz + 15^\circ = ip p$, and

4. $R. pi :: ip p. ip. \&c.$

IV. To make a declining inclining vertical Dial, decl. 25° incl. 40° .

Fig. 69.

SEt off $6d = 25^\circ$, join dd , draw $dx d$, so as rhe $L^c sda = 40^\circ$. So $dx d$ represents the inclining declining Plan, whose Pole is y , zzz the Perpendicular to the Plan, ax the Distance of the Perpendicular from the Line of 12 $y p m$ y is the Substilar, am its Distance

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Distance from 12, and xm its Distance from the Perpendicular. ds is the Co-Declination, $ab, ac, ae, \&c.$ are the Hour-Distances from 12, and $xa, xb, \&c.$ the Hour-Distances from the Perpendicular.

In the $\Delta^a ads$. R. $'ds :: 'ads. 'as$
 $'as. R :: 'ds. 'das = Pam$
 $\cdot 31.P.4.T. 'ads. 'as :: R. 'da.$

$90^\circ - da = ax$ Dist. Line 12 from Per.

In $\Delta^b Pam$. R. $'pa :: 'Pam. 'pm$ elev. Stile

$'pm. R :: 'pa. 'sam$ Dist. Subst. Mer.

$'pm. R :: 'am. 'apm$ Incl. Mer.

In $\Delta^c pmb$. R. $'pm :: 'mpb.$

$(mpa + apb) 'mb, \&c.$ for the Hour-Distances,

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The Analemma:

O R,

Orthographick Projection of the Sphere.

Fig. 70.

THIS Projection supposes the Eye in the Axis of the Primitive, and at an infinite Distance: So that all right Circles, whether lesser or greater, are projected into right Lines, and are measur'd on the Line of Sines.

Let the Primitive be the solstitial Colure EQ the Equator, pp its Axis, CL the Ecliptick, $C\infty$, $L\wp$ the Tropicks AR , NT , the Polars; on the Ecliptick, set off $\nu\delta = 30^\circ$ (the Sun's Longitude *Apr. 11*) draw $m\delta o \parallel QE$ cutting the Horizon HR at s ; so mo is the Parallel of

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of Declination $m Q = v d$ the Declination; $m s$ the Semi-diurnal Arch, $s o$ the Semi-nocturnal, φd the right Ascension; φs the Ascensional Difference; $s f$ the Distance from Due East or West when rising; $v f$ his Altitude when due East or West; and $v s$ his Amplitude.

Suppose $H x$ the Altitude, draw $x y \parallel H r$, cutting $z n$ the prime vertical at ϕ , the $\parallel$ of Declination $m o$ at σ and the Circle of 6 , $p p$ at δ , then σ shall be the Sun's Place in the Analemma at that Time; σd the Sun's Distance from 6 ; 6ϕ his Distance from East or West, and σy his Azimuth from North.

To reduce any Arch of a lesser Circle as $d s$ to the like Arch of a greater.

Draw $v o$, join $s \pi \parallel d v$, 'tis plain that $d o. v o :: d s. v \pi$.

APPEN-

APPENDIX.

Applying the Doctrine of plain
Triangles to taking Heights
and Distances, and to Plain
and Mercator Sailing.

HAVING already illustrated
the Doctrine of Spheric
Triangles in the Solution
of the Problems of the
Sphere by Calculation; we shall now
apply plain Trigonometry to the taking
of Heights and Distances, and to plain
and mercator Sailing.

In order to our laying down on Paper
the Triangles to be solved; we must first
have a Line of Chords, which is already
explain'd.

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2. A Line of equal Parts as AB , where, if the Divisions from c to B be Units, then those from c to A shall be Tens; but if the Divisions betwixt c and A be reckoned Hundreds, then these from c to B shall be Tens, &c. a Line Divided in this Sort Sailors call a *Line of Leagues*.

Draw $cd \perp AB$, and divide it into 10 equal Parts, through the several Divisions draw Lines $\parallel AB$ and complete the rectangle $ABGF$, divide FG as AB is divided; from c to the first Division in dG draw ce , and through each Division in cB draw Lines parallel to ce , these shall cut off the Units on the parallel Lines $k1, l2$, &c. for 10. $10 :: cd. de :: c9. 99 :: 9. 9.$ &c.

A Quadrant is the 4th Part of a Circle DK divided into its 90° , and furnished with Sights n, o , and a Plum-Line LG fixt to the Center L .

In looking thro' the Sights of the Quadrant to any Height the $L^e KLG$ (= Arch KG cut off by the Plum Line LG) is equal to the $L^e ADF$ form'd by Two Rays

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Rays from the Eye to the Height, one to the Top of the Height, the other parallel to the Horizon. Draw the Horizontal Line MLE the Plummets G falling perpendicular to the Horizon $MLG = 90^\circ = DLK$ and $MLD (= ALE) = MLG - DLG = DLK - DLG = GLK$.

A Graphometer is a Semi-Circle NDB divided into 180° . It has Two Sights x, o , on its Diameter & other Two n, z , on a Ruler that moves on its Center c. It is plain, that (the Graphometer being so placed that any Point B is seen thro' the Sights o, x , and the Ruler being so placed, that any other Point A may be seen thro' the Sights z, n), the Arch ND is the Measure of the Angle ACB. Fig. 74.

Probl. I. Fig. 73.

To take any accessible Height AB on a Horizontal Plan.

IN the $\triangle^{\text{le}} ADF$ right angled at F, $FD = NB$ may be measur'd; $L^{\text{le}} ADF$ and consequently FAD is known

T 2

and

^a 1. P. 3.

Trig.

^b 2. P. 3.

Trig.

and $\text{FAD. FD}^2 :: \text{FDA. FA}$
 $\text{OR R. DF}^b :: \text{ADF. AF, AF} \downarrow$
 $\text{FB the Observator's Height}$
 $= \text{AB.}$

Probl. II.

Fig. 75.

*To take any inaccessible Height AB,
 or any Part of an inaccessible
 Height as AC. or CB.*

AT any Station D take the Angles
 $\text{ADB, CDB; retire to any Di-}$
 $\text{stance DE take the Angle AED. So}$
 $\text{the Angles EAD, DAB, BCD, EDA,}$
 DCA are known and

$$\text{EAD. ED}^2 :: \text{E. AD}$$

$$\text{R. AD}^2 :: \text{ADB. AB}$$

$$\text{ACD. AD}^2 :: \text{ADC. AC.}$$

$$\text{AB} - \text{AC} = \text{CB}$$

Probl. III.

Fig. 76.

*To find the Distance of Two Places,
 A. B. of which one A is accessible.*

PLace your Graphometer at C, and
 take the Angle ACB, find like-
 wise

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wise the $L^{\text{ic}} CAB$ then $L^{\text{ic}} B$ is known,
measure AC . So $^{\text{s}}B. AC :: ^{\text{s}}C. AB$.
Q. E. F.

Probl. IV. Fig. 77.

*To find the Distance of Two Places
A, B, both inaccessible.*

Choise any Two Stations C, D mea-
sure their Distance CD ; with
your Graphometer, take the
Angles $ACB, ACD, BDA,$
 BDC ; the Angles $BCD,$
 CBD, ADC, CAD are
known

In the $\triangle BCD. ^{\text{s}}CBD. CD :: ^{\text{s}}BDC. BC$

In the $\triangle ACD. ^{\text{s}}CAD. CD :: ^{\text{s}}ADC. AC$

In the $\triangle ACB. BC + CA. BC - CA ::$

$\frac{1}{2} CAB + CBA. \frac{1}{2} CAB - CBA.$

So the Angles CAB, CBA are easily
known. In the $\triangle ACB. ^{\text{s}}ABC. AC ::$
 $^{\text{s}}ACB. AB. \text{Q. E. F.}$

The sensible Horizon represents a
Circle whose Plan is the Field on which
we stand, or the Surface of the Sea
expos'd to our View when at Sea, and
therefore

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therefore Sailors considering no more of this Earth but what they see have represented the Surface of this Earth, as a Plan on which they suppose all the Meridians to be parallel Lines, and the Parallels of Declination other Parallel Lines crossing the Meridians at right Angles, they divide both the Meridians and Parallels of Declination into equal Parts, and these equal Divisions on the Meridians represent the Degrees of Latitude, and the like Divisions on the Parallels of Declination represent the Degrees of Longitude ; the Rectangle form'd by these Meridians and Parallels thus drawn they call the *Plain Chart*, in which 'tis plain the Degrees of Longitude upon all Parallels are equal, a Grand Error.

Fig. 26. The Sea Compass is a Circle parallel to the sensible Horizon, & is divided into 32 equal Parts or Rumbs, & its plain these Divisions point out the like Divisions on the sensible Horizon, and therefore the Meridian Line being known by the Needle, the Point of the
Compass

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Compass on which the Ship steers, that is the Angle that the Ship's Way makes with the Meridian, is easily known by reckoning $11^{\circ} 15'$ to one Point.

The Ship Sailing to any Distance NP upon any Point of the Compass as SSE if from P to NS, the Meridian of the Place at which you set Sail, a Perpendicular be drawn; there shall be form'd a right angled $\Delta^{lc} N S P$ in which the $L^{lc} N$ is the Course; NP the Ship's Way or Distance run; NS the Difference or Change of Latitude; PS the Departure: In that Δ^{lc} , the Course and Distance run being given, the Difference of Latitude and Departure are easily found by plain Trig. For R. NP :: $^{\circ}P$. NS :: $^{\circ}N$. PS.

Fig. 78.

Or thus:

Drawing a Line equal to the Chord of 90° that Line shall represent 8 Points of the Compass, and taking from the Line of Chords the Degrees answering to the several Points and Quarter Points and setting of the Chords of these Degrees on the Line of 8 Points, you shall have

have made a Line of Rumbs, by the Help of which Line, and the Line of equal Parts, called the Line of Leagues, the Triangle may be describ'd thus:

Fig. 79. Suppose we saild SSE 240 Leagues, draw NS representing the Meridian of the Place at which you set Sail; upon the Center N, with the Chord of 60° , strike an Arch MD; from the Line of Rumbs, take of Two Points equal to SSE, set that from M to D, from the Line of Leagues take 240, with that Distance upon the Center N, strike an Arch EP, join ND and continue it to P, draw PS LN, so NS is the Difference of Latitude, and PS the Departure in Leagues measur'd on the Line of Leagues.

The same may be done by the Tables of Latitude and Departure thus: Find the Distance run in the Margin on the left Hand, and the Course on the Top or Foot of the Page tracing both, you shall at the Point of Concourse find the Difference of Latitude and the Departure.

To set off any Course. Fig. 80.

DRAW NS at Pleasure, representing the Meridian of the Place at which you set Sail, in it take any Point A for the Place it self; thro' A draw WE LNS, 'tis plain that, making N North, s shall be South, E East and w West, 'tis plain too that all the Points that are compounded of South and East shall fall betwixt s and E, and all compounded of South and West betwixt s and w, &c.

To draw a Traverse.

DRAW a Line representing the Meridian of the Place where you set Sail at first, and set off the first Course and Distance; parallel to your first Meridian draw the Meridian of the Ship's Place at the Beginning of the 2d Course, set off the 2d Course as before, and so on for 3d, 4th, &c. A Line joining the Points where you at first set Sail,

U

and

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and the Ship's Place at the End of the last Course gives the bearing of the 2 Places, and letting fall a Perpendicular to the first Meridian, that $\perp$ gives the Departure, and cuts off from the Meridian the Difference of Latitude.

Example. Fig. 81.

Sail'd from *London* in N. Latitude $51^{\circ} 30'$ S S E $\frac{3}{4}$ E 220 Leagues. Thence S by E $\frac{1}{2}$ E 180 Leagues, thence S by W 310 Leagues, thence N E 140, thence S by E 200 Leagues.

Or otherwise by the Traverse Table thus. For each Course and Distance find the Difference of Latitude and Departure by the Tables, the Difference betwixt the Sum of all the Latitudes Northward, and the Sum of all the Latitudes Southward gives the Difference of Latitude toward the Part of the greater Latitude. The same way the Difference betwixt the East and West Departures gives the Departure toward the Part of the greater.

Points

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Points	Course.	Distance.	Difference Latitude.		Departure.	
			SOUTH.	NORT.	EAST.	WEST.
2 $\frac{3}{4}$	SSE $\frac{3}{4}$ E	220	188.7		113.0	
1 $\frac{1}{2}$	SbE $\frac{1}{2}$ E	180	172.3		52.2	
1	sbW	310	304.0			60.5
4	NE	140		99.0	99.0	
1	SbE	200	196.2		39.0	
			861.2		303.2	
			99.0		60.5	
			762.2		242.7	

To reduce Leagues to Degrees of Lat.

Divide by 20, the Quot gives Degrees, the Remainder multiply'd by 3 gives Minutes.

To reduce Miles to Degrees.

Divide by 60, the Quot gives Degrees, the Remainder Minutes.

To find how many Miles answer to 1° of any Parallel of Latitude, supposing 60 Miles to answer to 1° of the Equator, say as R. 60 :: σ Latitude. Number of Miles answering to 1° of the Parallel of that Latitude. Fig. 82.

FOR making DBC the Equator, whose Radius AB , and dbc the $\parallel$ of Latitude, whose Radius Ab , making BC 1° of the Equator, 'tis plain, Circles being similar Figures, bc is 1° of the $\parallel$ of Latitude, and $AB. BC :: Ab. bc$ i. e. R. 60 :: $Ab. bc = 1^\circ$ of the $\parallel$ of Latitude, but Ab is $= \sigma$ Latitude :

For making PEP the Meridian, EQ the Diameter of the Equator, & la the Diameter of any $\parallel$ of Latitude, 'tis plain le the Radius of that $\parallel$ is the Sine of lp the Co-Latitude. Ergo, &c.

Hence, the Parallels of Latitude decrease in the Ratio of the Radius to σ Latitude.

The

APPENDIX. 157

The Parallels of Latitude reckon'd from the Pole toward the Equator increase in the Ratio of the σ Latitude to the Radius that is (by 11th *Prop.* Part I. *Trig.*) in the Ratio of the Radius to the Secant of the Latitude. Therefore Mr. *Wright*, considering the Error in the plain Chart, where the Degrees of Longitude are made equal in all Latitudes, has corrected the Sea-Chart thus: He allows the Meridians to be $\parallel$ still, and consequently the Degrees of Longitude to be every where equal as the Degrees of Latitude are on the Natural Globe, but he augments the Degrees of Latitude on the Sea-Chart in the same Ratio that the Degrees of Longitude increase on the natural Globe, reckoning from the Pole towards the Equator, to wit, in the Ratio of the Radius to the Secant of the Latitude, and these several Degrees of Latitude he expresses in Parts, of which 1° of Longitude contains a certain Number, suppose 60, he calculates Tables of these Degrees, which he calls *Tables of Meridianal Parts*.

To

To find the Meridional Parts answering to any Difference of Latitude.

I. **I**F the Latitude be reckon'd from the Equator North or South, you'll find the Meridional Parts answering to it in the Tables.

II. If both Latitudes be on the same Side of the Equator, the Meridional Parts answering to the greater Latitude — those of the lesser Latitude give the Meridional Parts of the Difference of Latitude.

III. If the Two Latitudes be toward contrary Parts, *viz.* one North and the other South, the Sum of their Meridional Parts gives the Meridional Parts of the Difference of Latitude.

I. *To find the Ship's Place on the Mercator-Chart, the Course and Distance being given, suppose a Ship*

APPENDIX. 159

Ship Sails SSW 240 Leagues from N. Latitude $51^{\circ} 30'$. Fig. 84.

Draw the $\Delta^{\text{c}} ABC$ as directed in the plain Chart, find AB the Difference of Latitude in Degrees, find the Meridional Parts answering to that Difference of Latitude, set off these Parts from A to D continue AC till it meet the IDE at E , so the Point E shall be the Ship's true Place in the Mercator Chart.

2. *Both Latitudes of Two Places A, C , and their Difference of Longitude being given, to find the Course and Distance.* Fig. 85.

IN the Triangle abc . ab represents the proper diff. Lat. bc the Departure, ac the Distance, $L^{\text{c}} A$ the Course, and $Ac b$ its Complement.

In the Triangle ABC . AB is the Meridional Difference of Lat. BC the Longitude, $L^{\text{c}} BAC$ the Course, and $L^{\text{c}} C$ its Complement, Reduce Diff. Long. BC to Miles or Minutes.

AB

160 APPENDIX.

A B. R :: B C. 'A the Course, reduce A b to Miles or Minutes, then 'A c b. a b :: R. A c the Distance.

3. Both Latitudes and the Course given, to find the Dist. and Diff. of Long.

'A c b. A b :: R. A c the Distance.

R. A B :: 'A. B C the Diff. Long.

4. Both Latitudes and the Distance given, to find the Course and Diff. of Long.

A c. R :: A b. σ A. the Course.

R. A B :: 'A. B C. the Diff. Long.

5. One Latitude, the Course and Difference of Longitude given, to find the other Latitude and Distance.

'A. B C :: 'A C B. A B the Meridional Diff. of Latitude, so the other Latitude is easily found, and consequently the proper Difference of Latitude A b.

'A c b. A b :: R. A c the Distance.

If these Cases be well understood, any other that can be proposed will be very easy.

For oblique Sailing, the Two following Examples shall suffice.

I. Two Ships sail from the same Port A, one sails N b E 120 Miles to B, the other N N W 188 Miles to C. I demand their Distance, and the bearing of both Ships from each other.

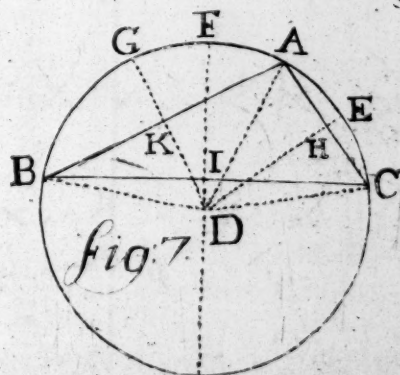
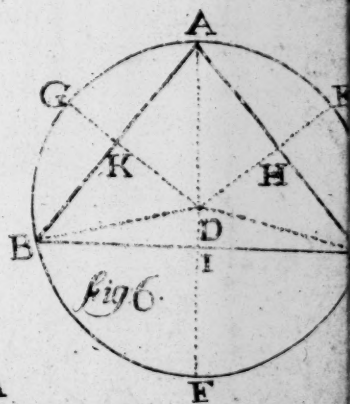
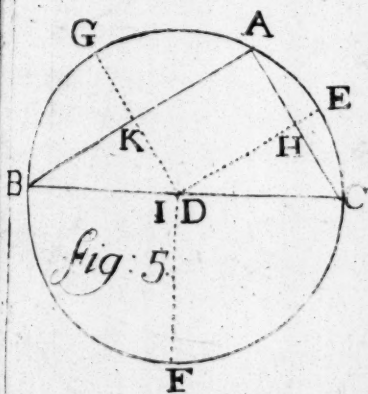
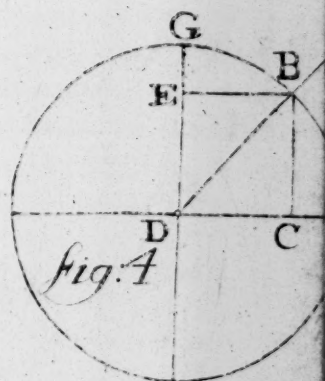
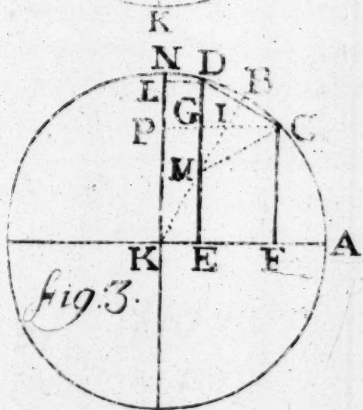
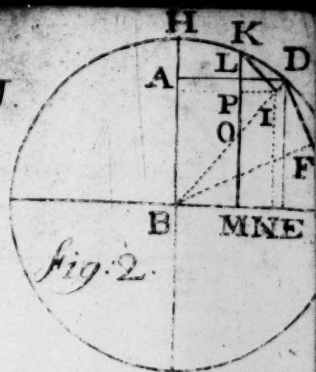
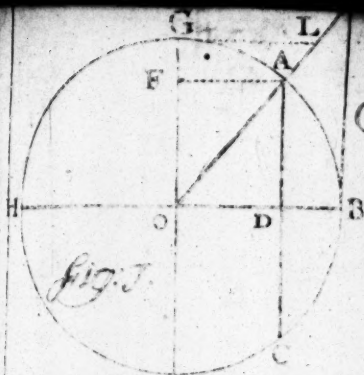
II. Two Islands K, M, have the same Longitude, and are distant 38 Miles; a Third Island L is distant from M 60 and from K 80 Miles. I demand the bearing of L from M and K.

The Problems already laid down sufficiently illustrate the Doctrine of plain Triangles, which was all I proposed.

F I N I S.



Plate J.





C

C

C

Fig

A

Plate 2

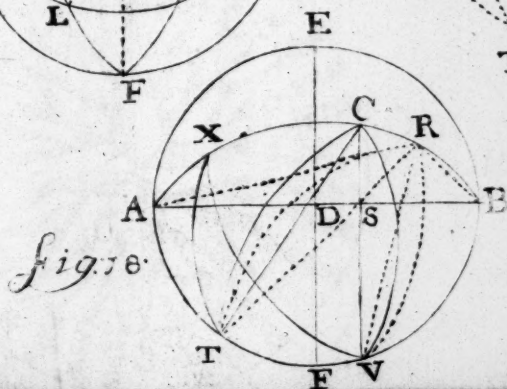
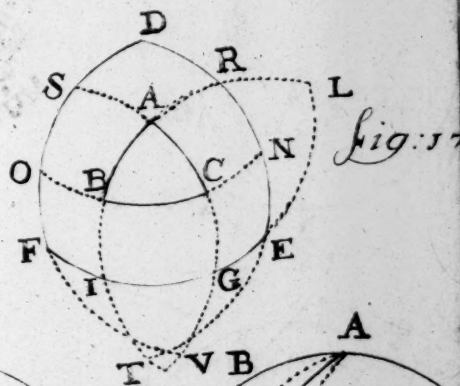
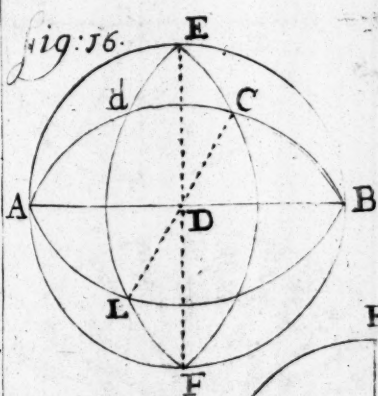
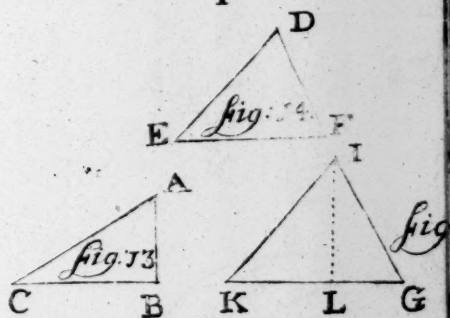
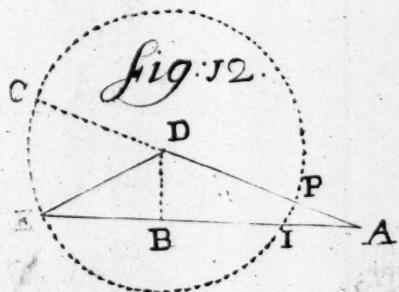
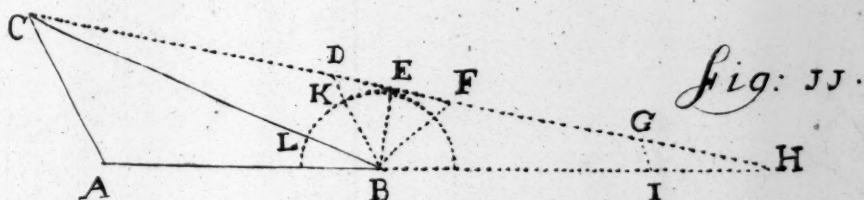
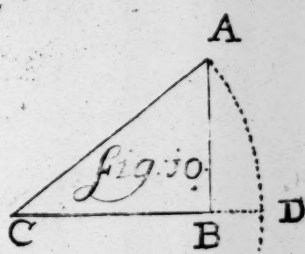
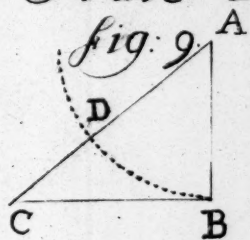
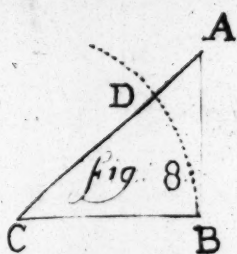




Plate 3c

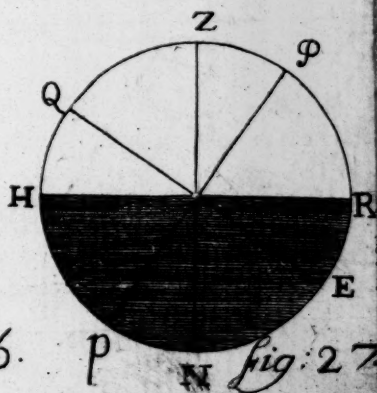
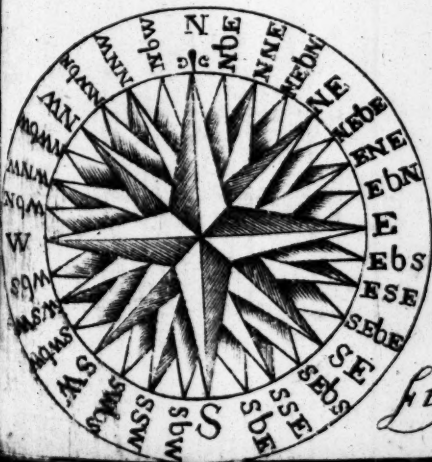
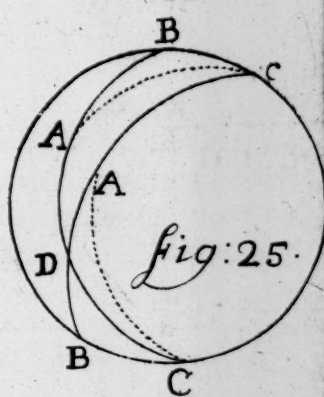
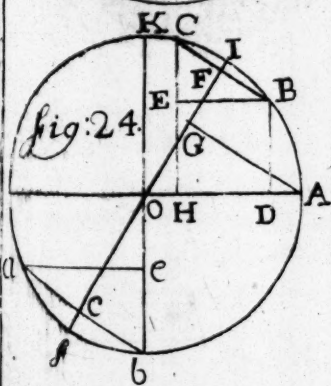
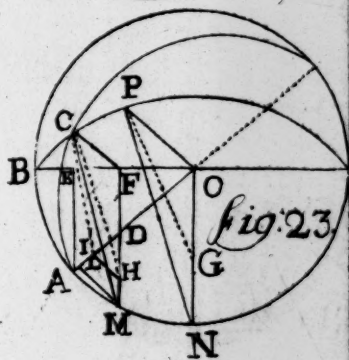
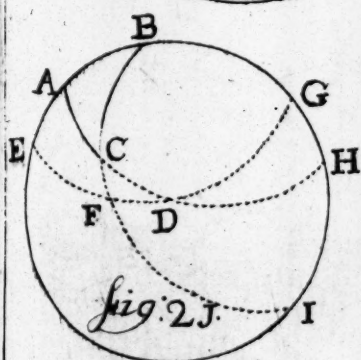
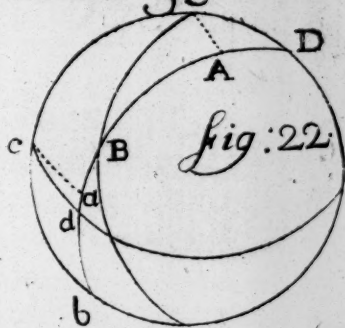
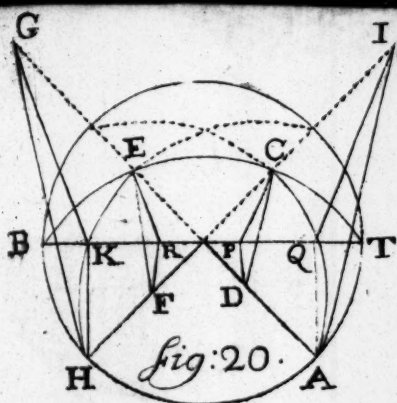
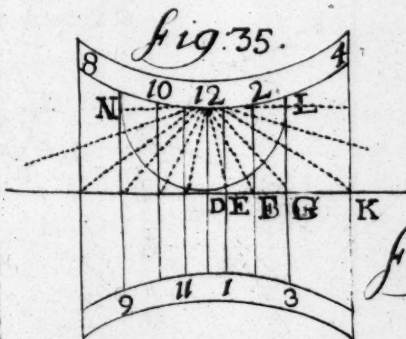
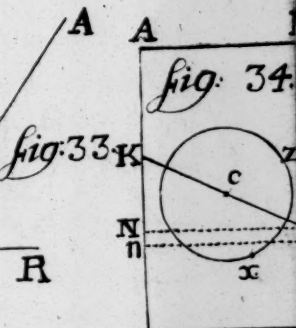
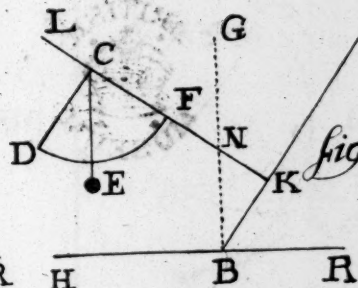
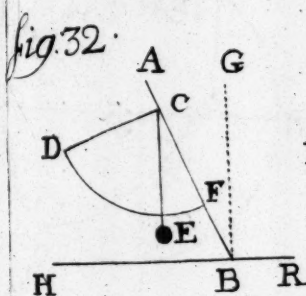
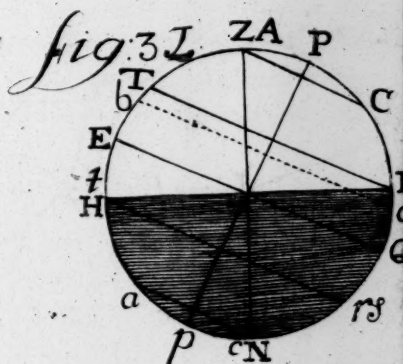
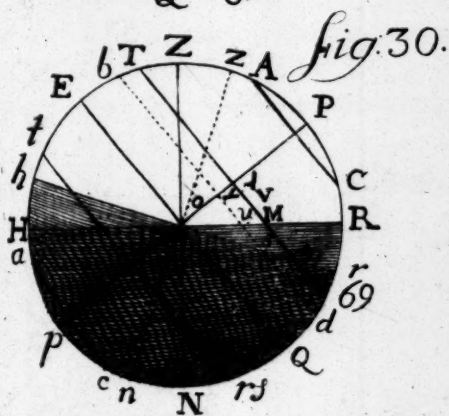
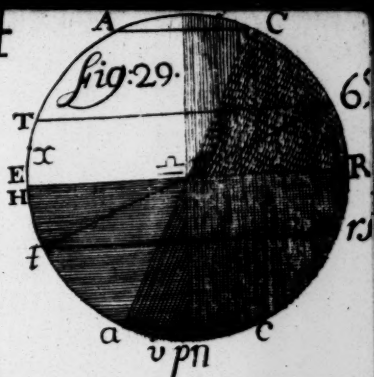
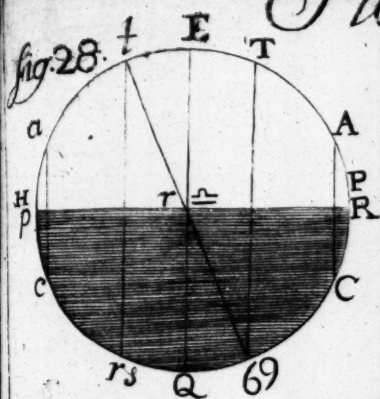


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Plate 4





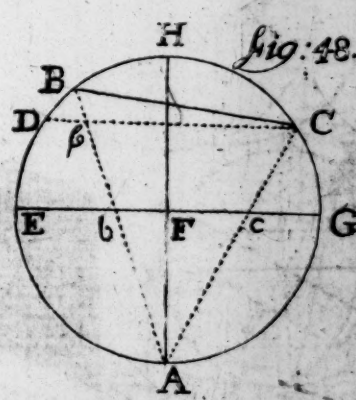
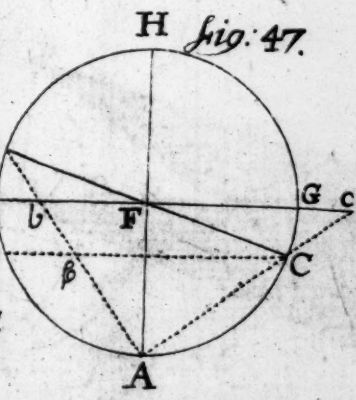
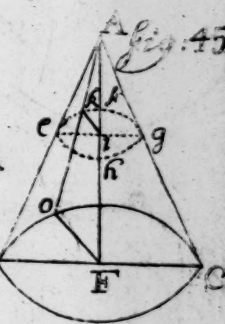
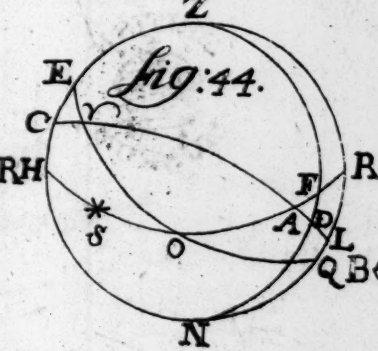
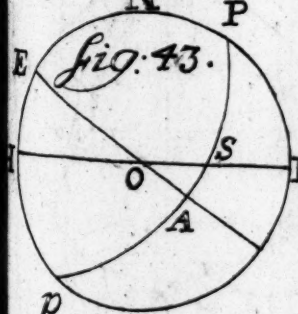
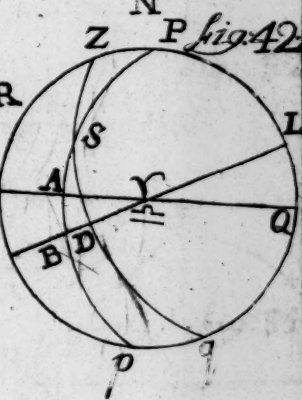
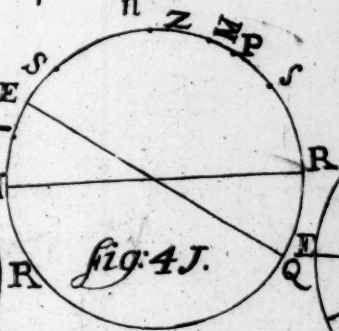
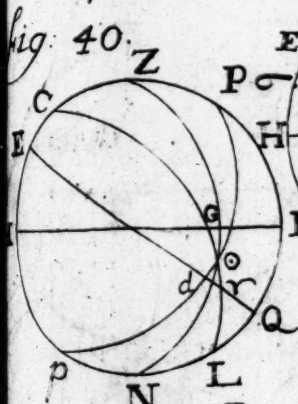
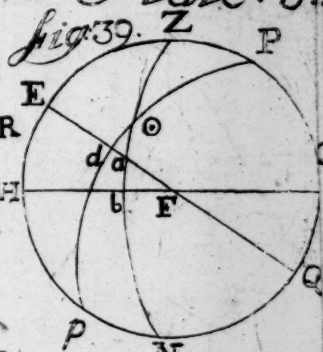
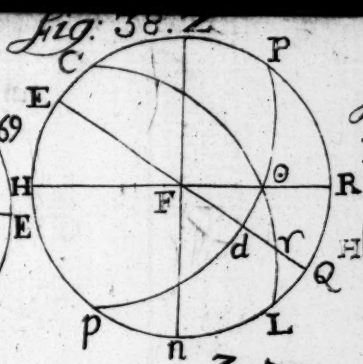
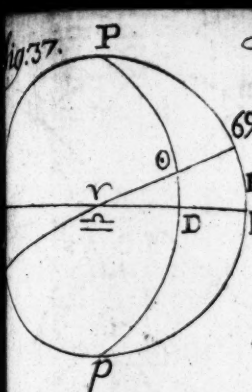
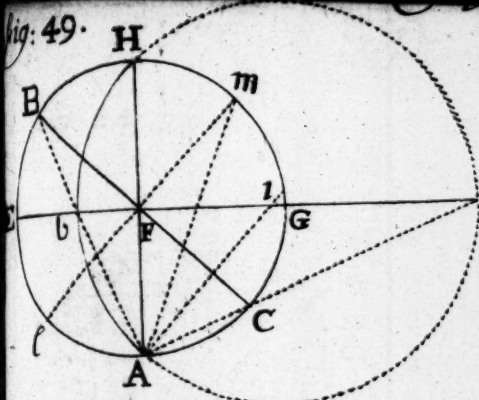




fig: 49.



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fig: 50.

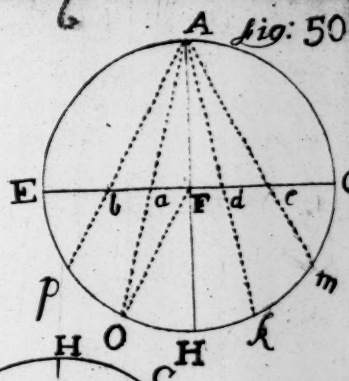


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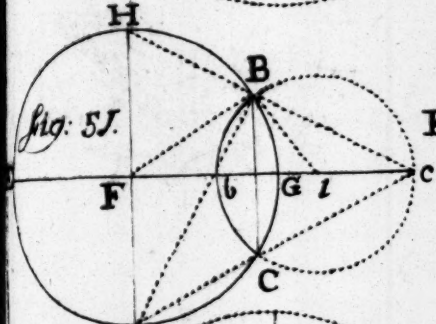


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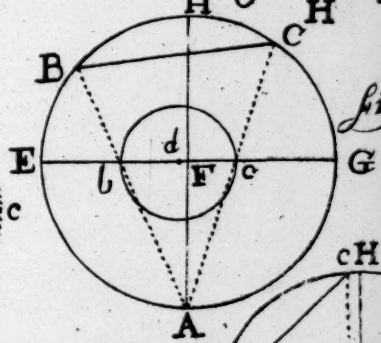


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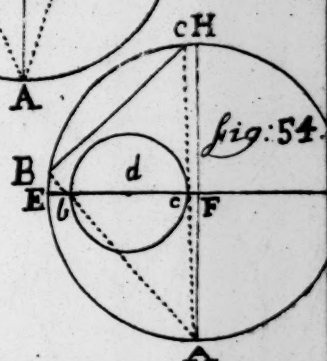


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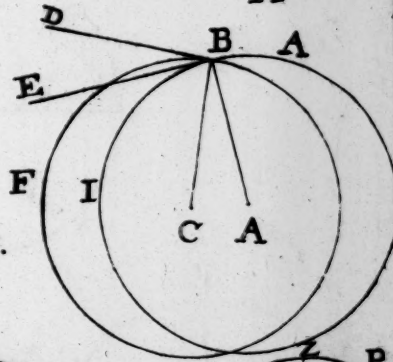


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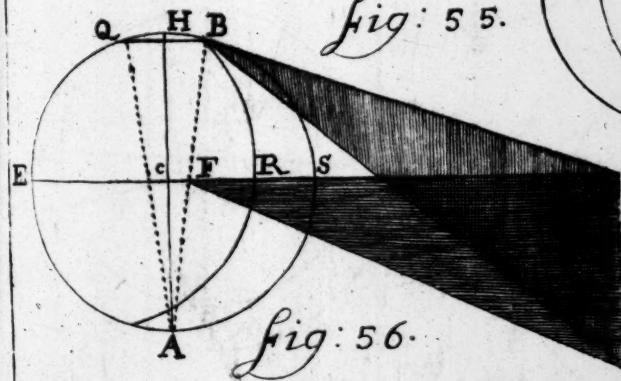


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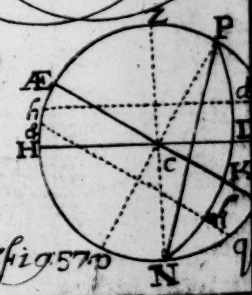




Plate. 7.

fig. 58.

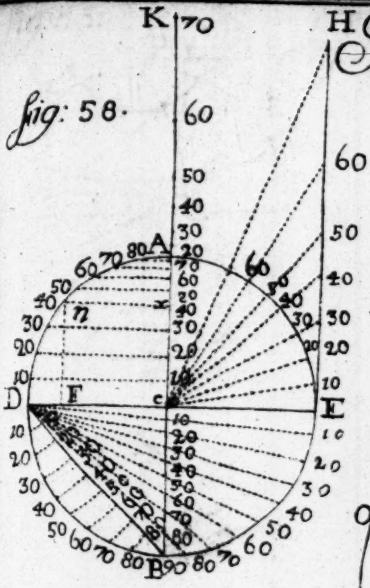


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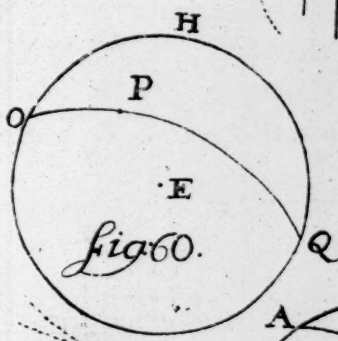
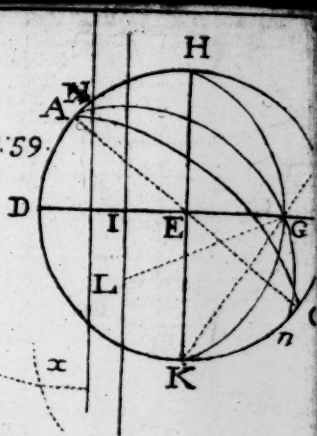


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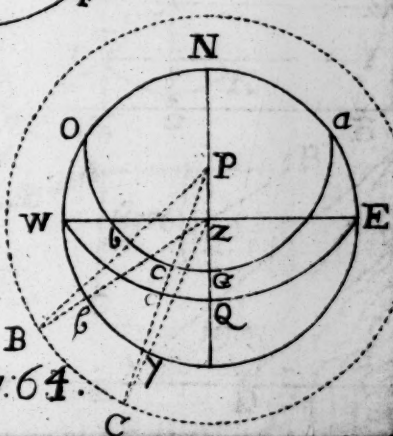
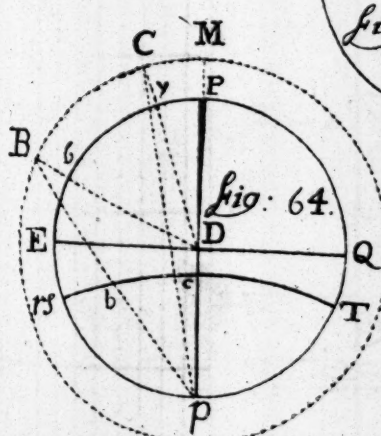
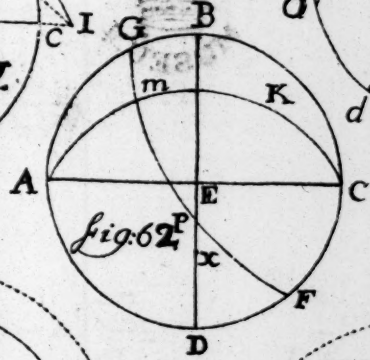
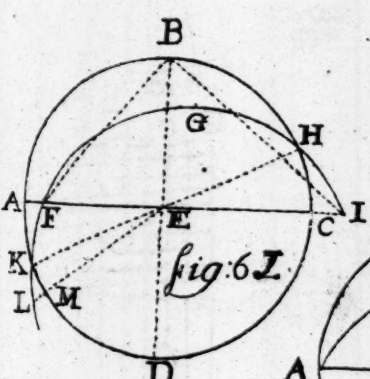
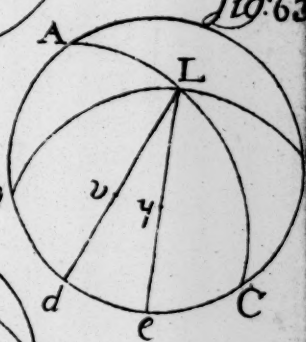


fig. 64.



Fig. 6



Fig. 6



Plate 8.

fig:65.

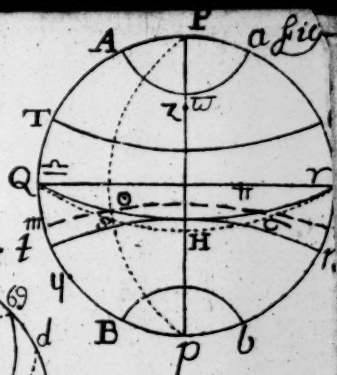
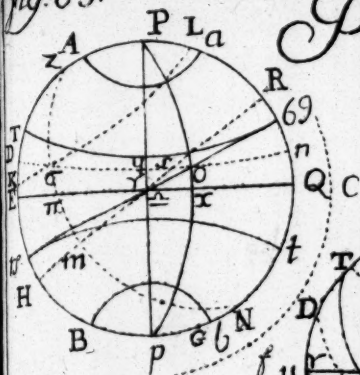


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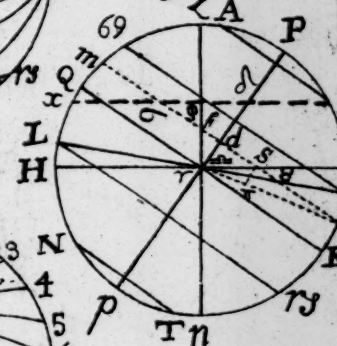


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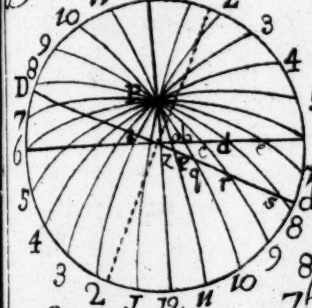


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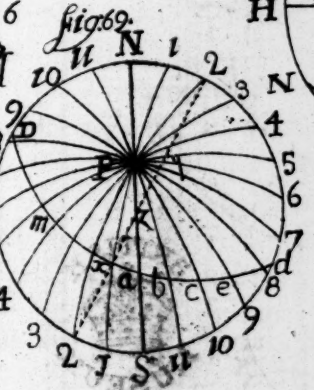


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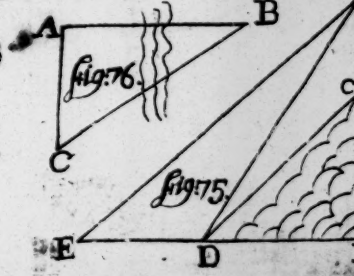
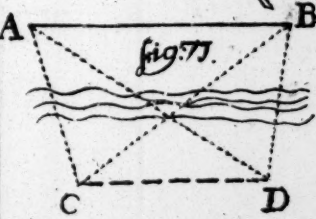
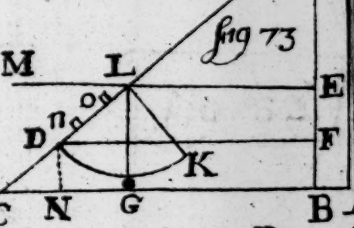
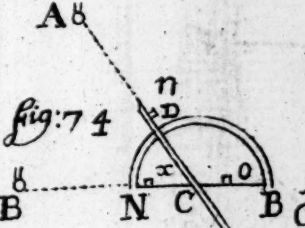
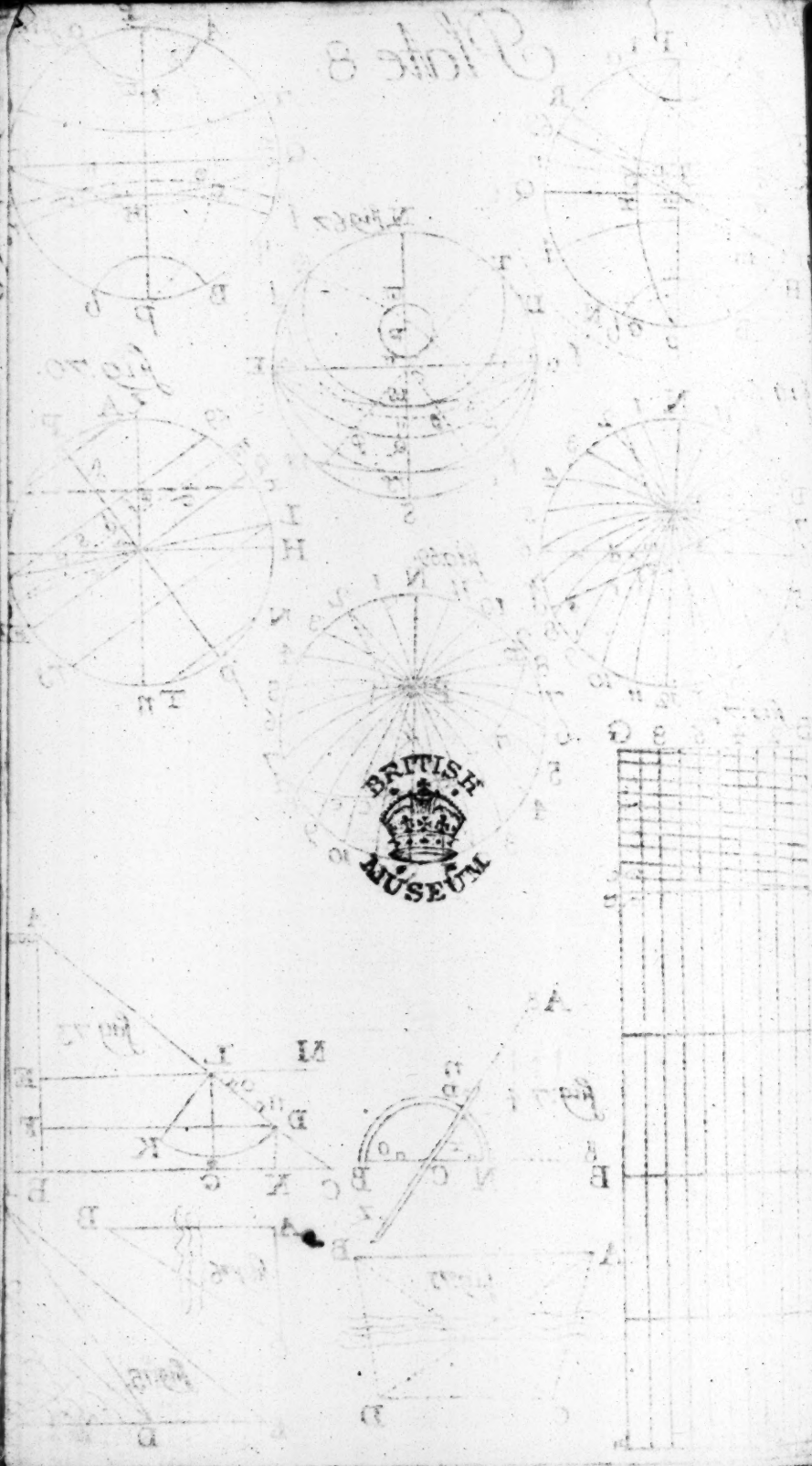
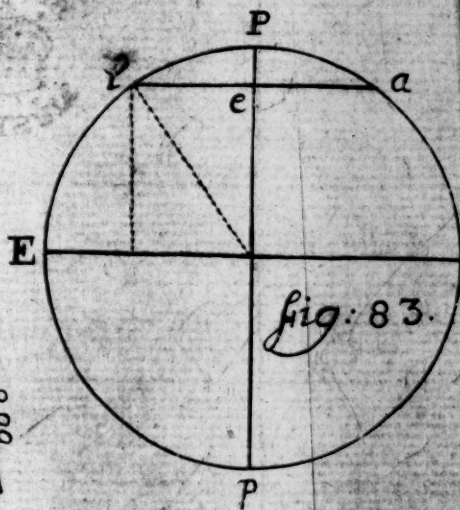
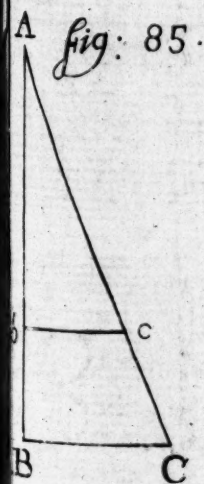
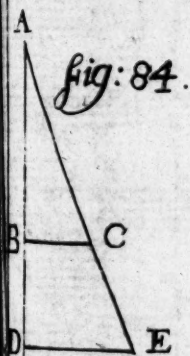
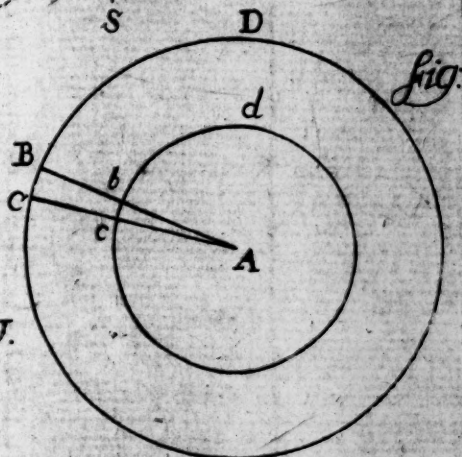
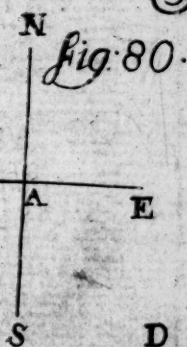
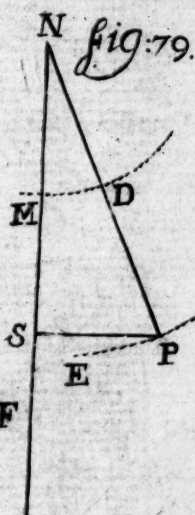
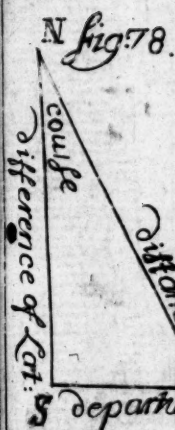


Plate 8



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Difference of Lat.





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fig: 80

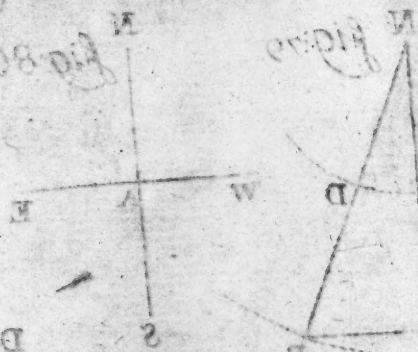


fig: 81



fig: 82



fig: 83

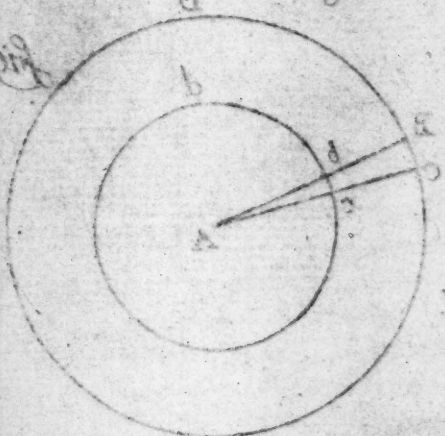


fig: 84



fig: 85



fig: 86



fig: 87

